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Optimizing maintenance decisions in railway wheelsets – a Markov Decision Process approach

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Abstract

The present dissertation models the decision problem of maintaining railway wheelsets as a Markov Decision Process (MDP), with the aim to provide a way to support condition-based maintenance for railway wheelsets. A brief background on the railway industry and the role of the railway wheelsets is provided, as well as some background on the technical standards that guide maintenance decisions. A practical example is explored with the estimation of Markov transition matrices (MTMs) for different condition states that depend on the wheelset wheel diameter, its mileage since last turning (or renewal) and damage occurrence. Bearing in mind a different set of possible maintenance actions, an optimal strategy is achieved, providing a map of best actions depending on the current state of the wheelset.

Keywords: Railway maintenance, railway asset management, maintenance optimisation, maintenance modelling, condition-based maintenance, probabilistic methods.

Resumo

A presente dissertação aborda o problema de decisão relativo à manutenção de rodados ferroviários como um Processo de Decisão de Markov (MDP), com o objetivo de fornecer uma ferramenta de auxílio às decisões de manutenção em relação aos rodados ferroviários. É feito um breve enquadramento da indústria ferroviária, o papel dos rodados na mesma e são dadas informações acerca das normas que regulam as decisões de manutenção dos referidos rodados ferroviários. Um exemplo prático é explorado através da estimativa das matrizes de transição de Markov (MTMs) para diferentes estados da roda, os quais variam de acordo com o diâmetro da roda do rodado, da sua quilometragem desde o último torneamento (ou renovação) e da ocorrência de dano. Dependendo do estado atual do rodado e, dentre um conjunto de ações possíveis de manutenção, é alcançada uma estratégia ótima, que fornece um mapa com as melhores decisões.

Palavras-chave: Manutenção ferroviária, gestão de recursos ferroviários, otimização da manutenção, modelação da manutenção, manutenção com base na condição, métodos probabilísticos.

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Acronyms

BS – British Standard

MCR – Markov chain with rewards

MDP – Markov Decision Process

MTM – Markov transition matrix

NDT – Non-destructive testing

POMDP – Partially Observable Markov Decision Process

UIC – International Union of Railways

Symbols

Latin symbols	**Definition**
a	Action
D	Wheel Diameter
d	Decision
F_h	Flange height
F_t	Flange thickness
I	Identity matrix
i	Index
j	Index
k	Index
mst	Mileage since last turning (or renewal)
n	Epoch
P	Probability transition matrix
p	Transition probability
P_1	MTM for the $\mathrm{a} = 1$
P_2	MTM for the $\mathrm{a} = 2$
P_3	MTM for the $\mathrm{a} = 3$
P_{TD}	Sub-Transition probability matrix for the turning situation from damaged states
P_{TND}	Sub-Transition probability matrix for the turning situation from non-damaged states
P_W	Sub-Transition probability matrix for the wear situation
P_{Wnd}	Sub-Transition probability matrix for the wear situation not considering the probabilities of damage occurrence
q	Immediate reward
R	Expected reward
r	Discount rate
s	State
s'	Next state
v	Expected total reward
t	Time period or step
X_n	Random vector that represents a stochastic process in different epochs n

Greek Symbols	**Definition**
γ	Discount factor
Δ	Variation
θ	Probability variable

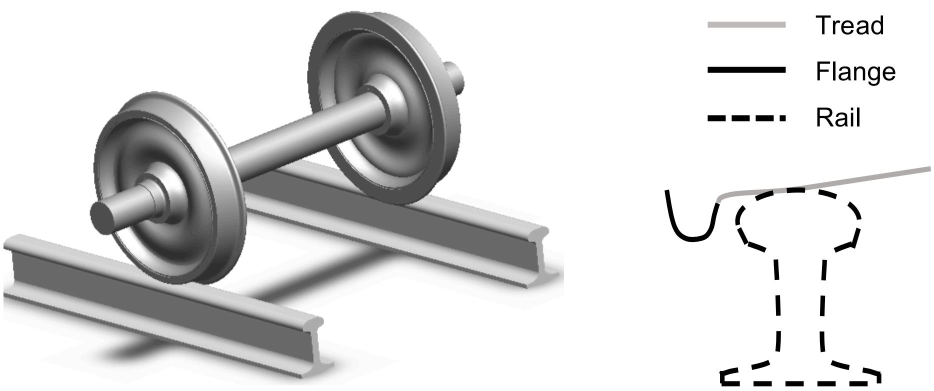
Chapter 1: Introduction

The aim of this chapter is to provide a general scope about the present dissertation: the main topics that are going to be covered, the methodology followed to analyse the data and the structure of the whole document. An introductory overview on the state-of-the-art and historical aspects of the railway wheelsets and railway industry is also provided.

1.1 – Railway Wheelsets

A train running along a railway track is a complex dynamic system, which combines many degrees of freedom of the free bodies, strict geometries of the wheel treads and rail heads, complex suspension mechanisms, non-conservative frictional forces and multiple dynamic force systems depending on whether or not the train is curving (Wickens 1998).

In the early days of the railway history, a single individual was responsible for all aspects of the design of railway wheelsets. The strength of the materials was a challenge, as well as the adhesion between the surfaces. The guidance of the vehicles was solved by the flanged wheel in the beginning of the 19^{th} century and the conicity of the rolling tread was also introduced to smooth the curves as well as to reduce the wear on the flanges (as it induces a differential effect on the wheelset when curving since the rail-wheel contact for each wheel happens in different zones of the cone with different diameters, thus, with different angular velocities on the surfaces).



(a) (b)

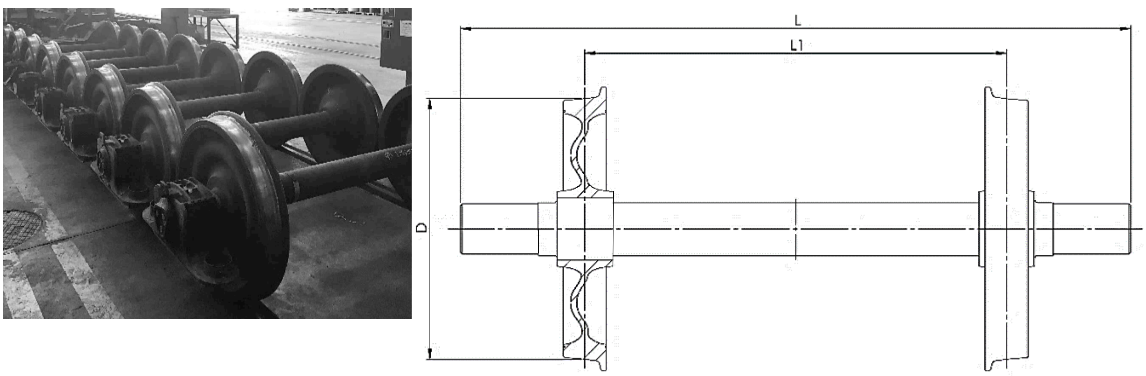
Figure 1.1: (a) Common railway wheelset, (b) wheel and rail interaction surfaces.

In the past, the design of the railway system was divided in two main parts: vehicles and their system of rolling stock were typically assigned to mechanical engineers, while the railway track to civil engineers. Meanwhile, with the technological development and the introduction of electrical/electronical components everywhere, the efficiency of processes and techniques became more important. This led to consider the railway-wheel surface, shape and interactions as a whole system, which needed to be optimized by getting together the path of the railway and the rolling mechanism of the railway-wheel (Iwnicki 2006).

As a consequence, inspection and measurement technologies, correction and production of the wheelsets and railway track have taken a big and essential role in the railway industry in order to guarantee the duration and reliability in an efficient way (Sharma 2016).

Nowadays, testing and instrumentation methods have become central to assure the accurate construction and validation of railway vehicle design, which should be tested in computer simulations, representing the dynamic behaviour of train vehicles and the railway track, and by adopting models based on tested data (Iwnicki 2006).

According to a technical standard from the International Union of Railways (UIC) on trailing stock/wheelsets UIC 510 – 2 OR (2004), a wheelset is composed by two wheels linked with a rigid axle. The axle should be a revolution piece made of rigid or hollow steel, whereas wheels shall be of two types: solid wheels or tyred wheels. The first type, made of rolled or forged steel, is the most common and will be the focus of the present study. The second one consists of separately mounted steel-wheel body, rolled, cast or wrought, coupled to a tyre mounted round the whole circumference of the wheel body without discontinuity and fastened with spring-clips[1].



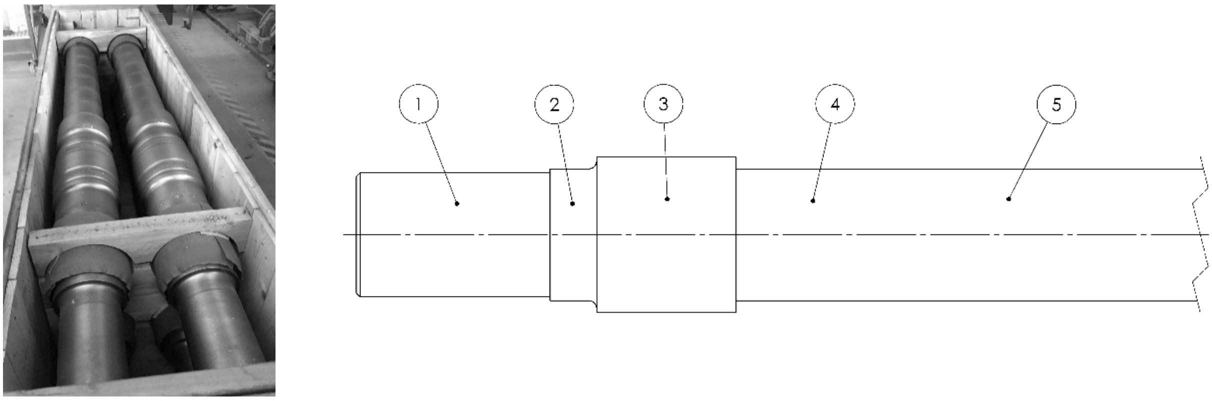
(a) (b)

Figure 1.2: Wheelsets: (a) with rigid axles and solid wheels, (b) a technical drawing representation.

The parts of a wheelset axle are (Figure 1.3):

1. Seat for axle box with bearings;
2. Abutment surface;
3. Wheel seat;
4. Transition zone and seat for brake disc, transmission or final drive;
5. Axle body.

[1] Today, tyred wheels are barely used in train industry and are being removed from service. Nevertheless, they are used in some metro systems to increase passenger comfort and avoid excessive noise.

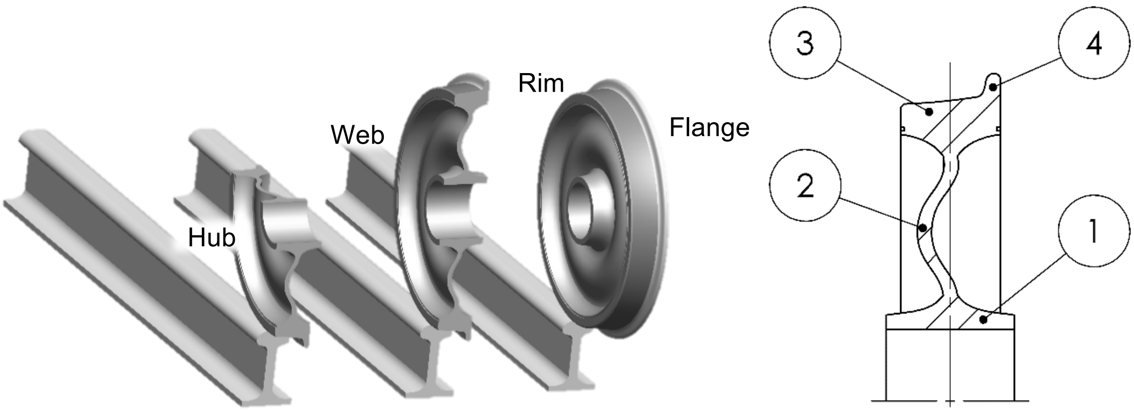


(a) (b)

Figure 1.3: (a) Rigid axles, (b) schematic representation of a standard axle.

A basic composition of a solid wheel has the following elements (Figure 1.4):

1. Hub: axle-wheel interface for a fit assembly;
2. Web: does the hub-rim connection as well as guarantees flexibility to the wheel;
3. Rim: rolling surface for the rail-wheel contact;
4. Flange: responsible for guidance and non-derailment of the wheelset on the rails.



(a) (b)

Figure 1.4: Functional elements of a wheel: (a) 3D representation, (b) section view.

1.2 – Research objectives and methodology

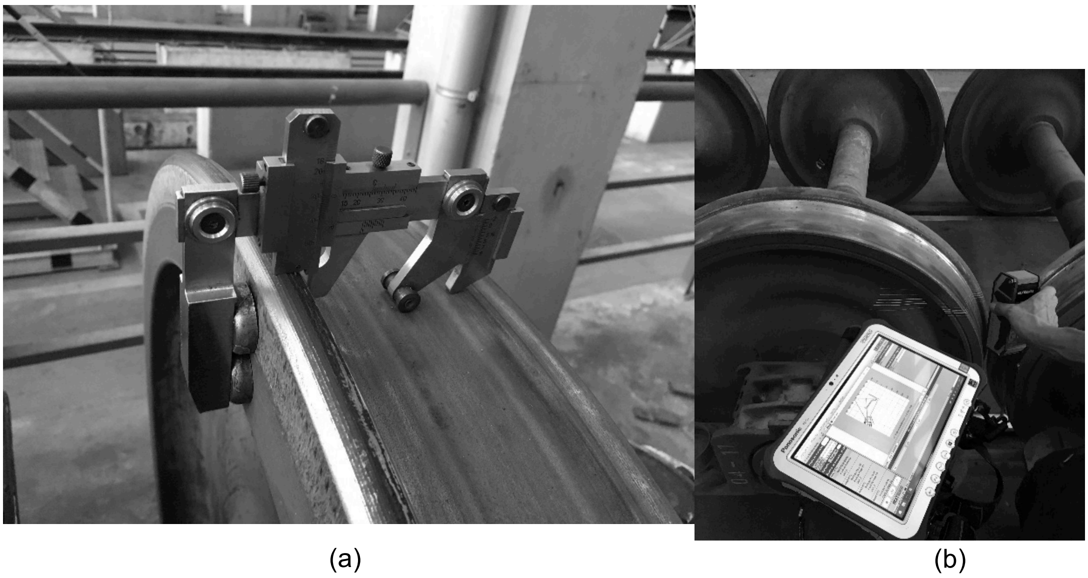
As the world population grows, the need for energy supply increases, resources become scarcer and with climate change, it has been an imperative guideline to create a strong and effective transport network provision. To face these challenges, it is advisable that the railway sector regain a larger share of transport mode choice, thus, the railway transport should be in a constant improvement to fulfil the needs of current and future users, and those of society in general.

Railway companies[2] search for the most economic, reliable and efficient processes to make their investments. In a competitive world, the railway companies try to conjecture innovative strategic plans

[2] Note that in the notion of railway companies, train operating companies and infrastructure managers are both included.

to manage their assets, employers and services. As one of the innovative examples, in terms of running gear, the next generation of light weight bogie systems will provide higher reliability and availability with lower maintenance costs. Less wear, damage, noise, vibration and energy loss are the results of optimized materials, new active suspensions and enhanced control technologies.

In the indicative list of priority research and innovation activities, there are the predictive, risk-based or condition-based maintenance systems. These policies should be built on cutting edge data mining from all relevant components of the rail infrastructure, including degradation modelling for whole-life cycle cost estimates and with a view to introduce non-destructive testing methods (Shift2Rail 2015). In the railway industry, train operating companies spend a significant part of their maintenance budget on wheelsets. As wheelsets take a critical role concerning the motion of the vehicles and the passenger comfort, their dimension must comply with tight standards for the wheel shape and diameter. On the other hand, due to their use and mileage, it is inherent to the wheel-profiles that they wear and damage, and so, inspection activities should monitor and control the evolution of the main indicators of degradation of a wheelset, and restore them if damage occurs and/or wear is higher than certain limits. The restoration of the shape of a wheelset can be scheduled within the preventive maintenance plan - planned actions - or in the corrective maintenance actions - remedial actions (Andrade and Stow 2016). Inspection activities have traditionally been executed through the use of a gauge device (Figure 1.5a), though they have slowly been replaced by laser equipment (Figure 1.5b).



(a) (b)

Figure 1.5: Inspection activities: (a) using a gauge device, (b) using a laser equipment.

Measurements taken with a laser equipment tend to be faster than with gauge devices, taking approximately 20 minutes for a multiple unit with 16 wheelsets, which compares with an average of 90 minutes for manual inspection with a gauge device. These time savings are easily converted in cost savings and have been the economic justification to adopt laser equipment. Moreover, gauge devices are more prone to human errors, thus, tend to have a lower precision than the laser equipment. The economic benefits of introducing such increase in precision are difficult to estimate, especially in the medium/long-term, given a certain maintenance strategy and maintenance yard constraints.

Therefore, the main research objective of the present study is to optimize maintenance decisions in railway wheelsets using a condition-based maintenance approach.

In order to achieve this goal, the following steps were pursued:

- Literature review on Markov Decision Process (MDP);
- Estimation of Markov transition matrices (MTMs) from published data;
- Model implementation in MATLAB® using a MDP Toolbox;

In a nutshell, the maintenance and renewal decision process will be modelled using a MDP approach and consequently optimizing maintenance decisions of railway wheelsets.

1.3 – Structure

The present dissertation is structured in five chapters. This first chapter introduced the main topic. Then, a brief and general presentation of the MDP is provided in Chapter 2 - the basic concepts/elements of Markov models and their decision processes are covered as well as their benefits. In Chapter 3, the technical details associated with the inspection, degradation and maintenance of railway wheelsets are explained and described, and in Chapter 4 a practical example is analysed, with the estimation of the MTMs, the definition of the reward/cost function and, finally, the optimal values are assessed, finding an optimal strategy of maintaining railway wheelsets. Lastly, Chapter 5 sums up all the analysis and main ideas referred during the previous chapters. It also identifies the limitations, simplifications and adaptations of the methods and procedures during the whole dissertation. It ends up with potential improvements, identifying future paths for further research.

Chapter 2: Markov Decision Process

Along this chapter, the main principles of the Markov Decision Process (MDP) and main ingredients are explained and integrated in a brief literature review of the application of these models in different scientific areas. Section 2.1 starts by discussing Markov models and the Markov property, then section 2.2 includes other ingredients/components to form the MDP.

2.1 – Markov models

Markov models have been introduced by the work of a Russian mathematician called Andrey Andreyevich Markov (1856 – 1922) in 1906 when, to prove his mathematical rival's hypothesis about the impossibility of future predictions wrong, he used a literary work of Alexander Pushkin (1799 - 1837), with a significant amount of words, to statistically analyse its letters and concluded what he sought out to validate: a correlation between vowels and consonants sequences in words. At that time, his study enabled the creation of more complex random generation systems than the probability models to describe Bernoulli (1654 – 1705) events such as flipping a coin. Curiously, Markov chains are still used nowadays in linguistics to find the author of a text (Vulpiani 2015).

Markov chains are specific mathematical models in stochastic processes that describe the evolution of a system that passes successively through different states. These systems can mathematically be modelled linearly, with matrices and vectors and abstractly represented as random paths in networks or graphs, to predict the long-term behaviour of the process (Sheskin 2016). In fact, when larger horizons of analysis are required, Markov Chains tend to be a better modelling choice compared to other state-of-the-art heuristic techniques for stochastic processes (Pathak et al. 2015).

A random process is a sequence of random events/variables. Following Sheskin (2016), a preliminary approach would consider time-scale separation (discrete time) and assume that the time interval between successive spaced points is constant. The spaced points are called epochs (n), and the time between them is called period or step (t). Although index n is most commonly used to represent a punctual observation in time, it can also indicate other parameters, such as the nth item inspected or the nth costumer served, with $n \in \{0, 1, 2, \dots\}$ and $t \in \{1, 2, \dots\}$ (Sheskin 2016).

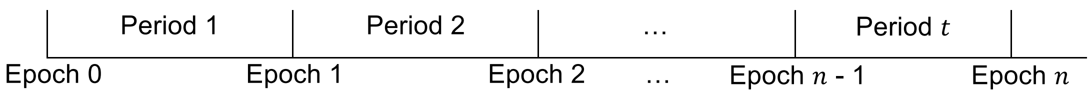


Figure 2.1: Sequence of consecutive epochs and periods.

For each n, X_n is a random vector ($X_n \in \mathbb{R}^r$) and it is called a stochastic process. A Markov chain has states (s) and it can have a finite or infinite number of states. All the Markov chains treated in this thesis have a finite number of states, $s \in \{1, 2, \dots, S\}$.

Uncertainty is inherent in the possible transitions from one state to another state from one epoch to the next one, and the entries of the vector X_n can represent the probabilities of the process being in each one of the states s at each epoch (or phase) n,

$$X_n = [P(X_n = 1) \quad P(X_n = 2) \quad \cdots \quad P(X_n = S)]. \tag{2.1}$$

The Markov stochastic process will respect the Markov property for stationary transition probabilities. This property states that the probability of the next state, conditioned on all history, depends only on (the probability of) current state, and not on all the past history of states visited before by the system. In stationary or homogeneous Markov chains, where the probability of going from one state to another is independent of the period at which the transition is being made, the Markov property can be written as follows:

$$P\ (X_{n+1} = i_{n+1} \mid X_n = i_n, X_{n-1} = i_{n-1}, \dots , X_0 = i_0) = P\ (X_{n+1} = i_{n+1} \mid X_n = i_n). \tag{2.2}$$

This conditional probability is called a transition probability and the transition matrix at epoch n is denoted as $P^{(n)} \equiv [p_{i,j,(n)}] \equiv P(X_{n+1} = j \mid X_n = i)$, whose entries are the probabilities that the system moves to state j in epoch $n+1$, given that in epoch n, the system was in state i, with $i,j\ \in\ \{1, 2, \dots , S\}$ (Yin and Zhang 2006).

At any given epoch n, the transitions between states can be depicted in an oriented network node graph as presented in Figure 2.2 and their probability values represented in a transition matrix, also called Markov transition matrix (MTM):

$$P \equiv [p_{i,j}] = \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} & \cdots & p_{1,S} \\ p_{2,1} & p_{2,2} & p_{2,3} & \cdots & p_{2,S} \\ p_{3,1} & p_{3,2} & p_{3,3} & \cdots & p_{3,S} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{S,1} & p_{S,2} & p_{S,3} & \cdots & p_{S,S} \end{bmatrix}. \tag{2.3}$$

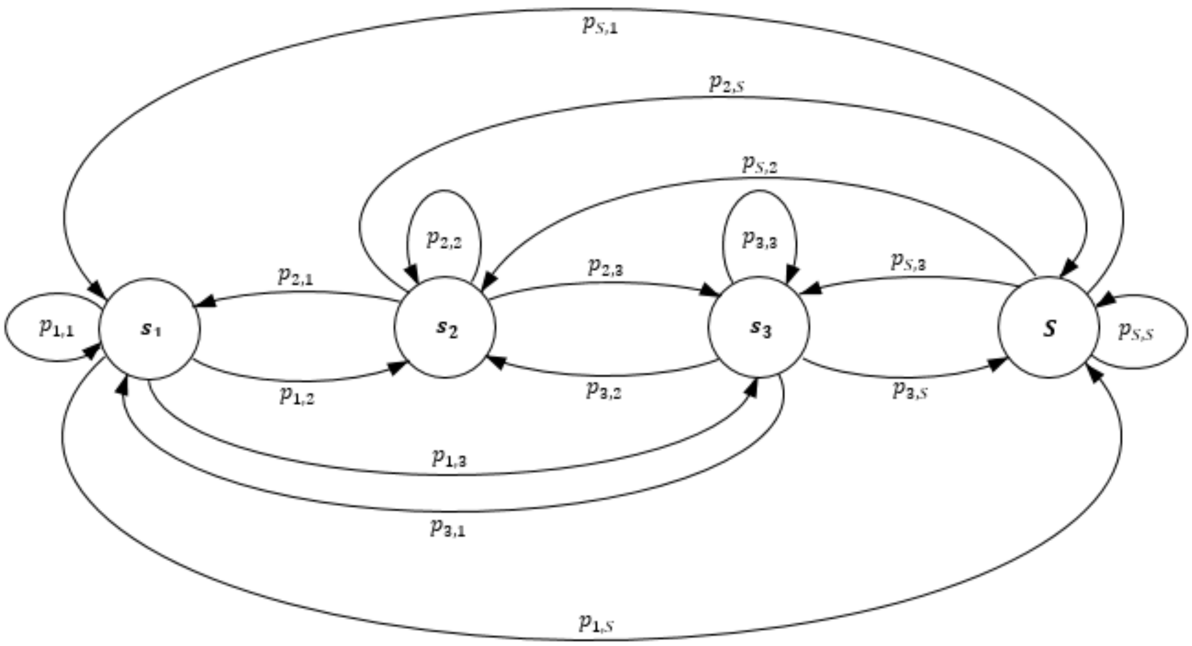


Figure 2.2: Transitions between states and corresponding probabilities.

A Markov matrix has all non-negative inputs and its values are:

$$0 \leq p_{i,j} \leq 1. \quad (2.4)$$

From Figure 2.2 and Equation (2.2), it is easily deducted that a transition from one state to another state (including staying at the same one) is a certain event, thus, the sum of all entries for a given row i in transition matrix $P \equiv [p_{i,j}]$ is equal to one:

$$\sum_{j=1}^{S} p_{i,j} = 1. \quad (2.5)$$

Given a stochastic process, let X_n be a row vector, whose entries are the probabilities that the process is at epoch n for each possible state. At the next epoch $n+1$, the process can evolve to different states and the row vector X_{n+1}, whose entries are the probabilities that the process is at epoch $n+1$ for each possible state, is obtained by the following equation (which only depends on the X_n immediately before and the transition matrix $P^{(n)}$):

$$X_{n+1} = X_n \cdot P^{(n)}. \quad (2.6)$$

Note that very often the transition matrix is stationary so it remains constant for every epoch[3]. Therefore, the succession of $\{X_n\}$ can be obtained as follows:

[3] I.e. $P^{(n)} = P$

$$
\begin{gathered}
X_0 \\
X_1 = X_0 \cdot P^{(0)} = X_0 \cdot P \\
X_2 = X_1 \cdot P^{(1)} = (X_0 \cdot P)\, P = X_0 \cdot (PP) = X_0 \cdot P^2 \\
X_3 = X_2 \cdot P^{(2)} = (X_0 \cdot P^2)\, P = X_0 \cdot (P^2 P) = X_0 \cdot P^3 \\
\dots \\
X_n = X_0 \cdot P^n.
\end{gathered}
\tag{2.7}
$$

With this result, one can easily conclude that in a Markov process, its future state at any time interval period can be predicted, if the transition matrix of the process is known and the row vector at epoch n. Let $\mathrm{k} \in \{0, 1, 2, \dots\}$ be the number of periods that have passed from the epoch n, the probabilities of visiting any state is given by the following equation:

$$
X_{n+k} = X_n P^k. \tag{2.8}
$$

This ease of access and computation of the probabilities of visiting future states and flexibility are the major advantages of Markov models (Sheskin 2016). In fact, they have been used in many applications in different scientific areas.

A few examples can be found below:

- Chemistry – modelling the evolution of chemical reactions in time (Kang 2014);
- Healthcare – helping in the treatment and prevention of progressive diseases (Jackson 2016);
- Meteorology – predicting the succession of weather conditions (Sheskin 2016);
- Migration of entities – being the basis to predict mobility of populations (Ignazzi 2015), study the motion of aggregated species or manage vehicle fleets (Ning and Sobel 2017);
- Communication networks and signal transmission – for finding probability distributions of queue size and message delays (Hayes 2013);
- Game theory – to every realization of a Markov process (each game's state theory) it is attached a payoff possible of being maximized or minimized according to the will of the player (Dresher 2016), it could also be used to optimize the loads of multi-player online game servers (Saeed et al. 2015);
- Internet search engines – modelling the page rank algorithm of the biggest search engine in the world (Lay 2012);
- Cognitive Science – playing a central role in the fields of machine learning, robotics and artificial intelligence (Ghahramani 2015).

2.2 – Markov decision process

A Markov Decision Process (MDP) is a controlled stochastic process in which a decision-maker is uncertain about the exact effect of executing a certain action, in the sense that, the system may transit to another state with a certain probability and visiting that state has a certain cost or reward (Papakonstantinou and Shinozuka 2014a). The goal is then to optimize the intended objective function (e.g. maximize the sum of all rewards or an average reward, or alternatively minimize the sum of all costs or average cost), over the set of solutions that are feasible for each state, supporting the decision-maker to take the best action at certain times/epochs in the timeline, and then preventing or limiting the deterioration of the objective (Gabrel et al. 2014).This optimization approach is in general made to advise decision-makers in sequential actions that must be performed based on actual data withdrawn from inspections or non-destructive tests, usually with the goal of minimizing life-cycle cost of the system (e.g. infrastructures). The set of actions that should be taken (usually depending on the state that the system is) is called a policy (Papakonstantinou and Shinozuka 2014a).

A MDP infers that for any time step (t), where the system is in a certain state (s), the agent takes an action (a) of a finite set of actions, $a \in \{1, 2, \dots, A\}$, and receives a specific corresponding reward (or cost) as a result of the chosen action. Each time the system visits state i at epoch n, a reward is earned. This reward is denoted as q_i.

In the majority of cases, the reward vector ($q_i \in \mathbb{R}^r$) is assumed to be stationary over time, similar to what happened with the transition probability $p_{i,j}$, and the reward vector represents the immediate independent rewards associated to the value of each process' state,

$$q_i = [q_1 \quad q_2 \quad \cdots \quad q_S]^T. \tag{2.9}$$

In Sheskin (2016), the vector of expected rewards received after k steps comes as:

$$R = P^k q_i. \tag{2.10}$$

Using probability theory, the scalar expected value for the total reward received after n epochs can be computed as:

$$E\left[R_{(n)}\right] \equiv \bar{R}_{(n)} = X_0 P^n q_i = X_n q_i. \tag{2.11}$$

As it was seen before, the MDP is a sequential decision process for which the decisions produce a sequence of Markov chains with rewards (MCRs). If decisions have to be taken to change in our benefit the natural path of the evolution of the Markov chains, there must be a series of possible actions allied to the process. The set of best actions to take for each of the possible states is called an optimal policy according to a given criterion. This rule is considered stationary over an infinite planning horizon and works together with the blocks of MCRs. The algorithms used to find an optimal policy are iterative approaches with dynamic actual data to calculate the gains of the MCRs in order to maximize or minimize them as intended. Due to the complexity of the process, for multi-state chains linear programming is imperative.

For each action in the process, a different change is caused in the odds of the next states, altering the transition probabilities from the actual state to the next one. As a consequence, a new dimension is added to the transition matrix. Denoting k as a possible action for each state, $p_{i,j}^{k}$ indicates the probability that the MDP will move to state j, at epoch $n+1$, starting from state i at epoch n, and knowing that action k is applied,

$$p_{i,j}^{k} = P(X_{n+1} = j \,|X_n = i \cap a_i = k). \tag{2.12}$$

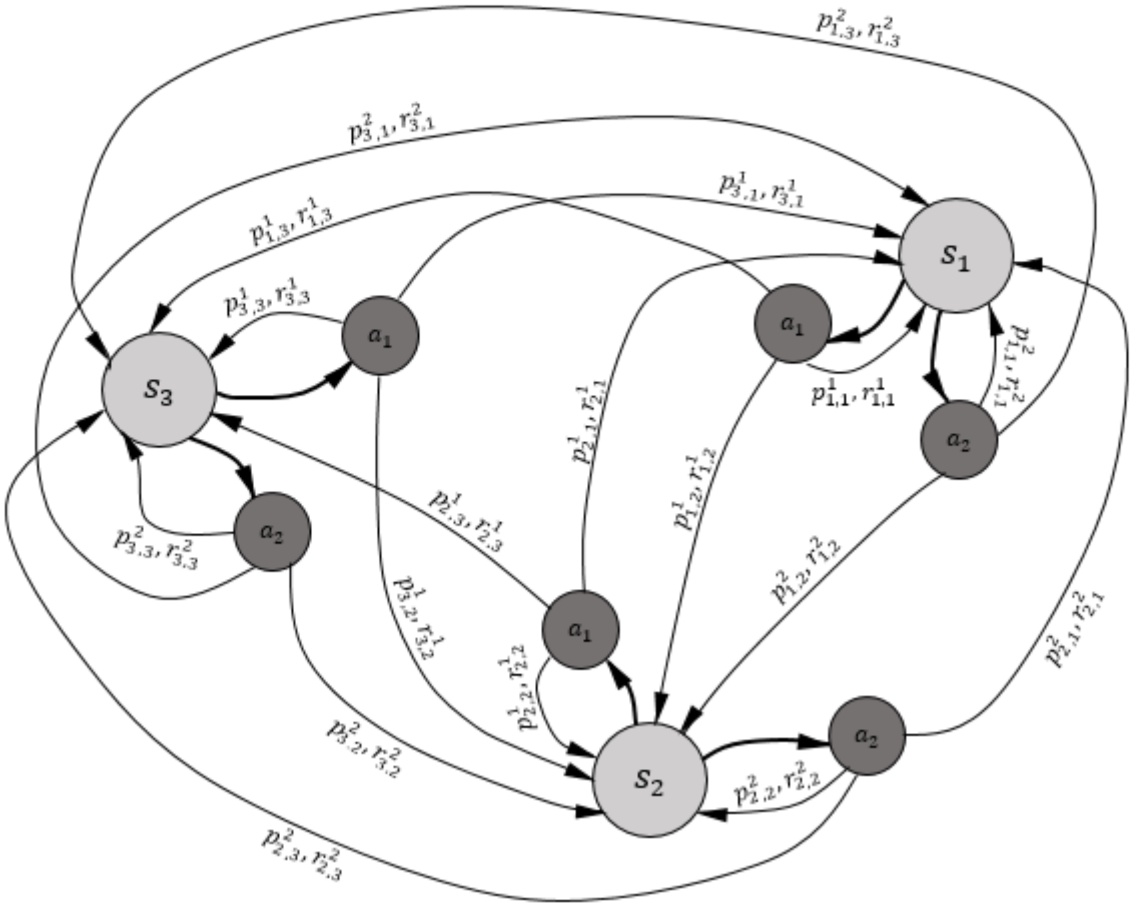


Figure 2.3: Example of a simple MCR with three states (s_1, s_2, s_3) and two actions (a_1, a_2).

Among the set of actions possible for each state, the chosen one, is called decision. The set of decisions for all states is called a policy and it is represented by the decision vector,

$$d_i = [d_1 \quad d_2 \quad \cdots \quad d_S]^T. \tag{2.13}$$

As written in Sheskin (2016), the entries of d_i indicate which is the best action to be made in every state. The best action is selected with the aid of a value iteration procedure that computes a vector of expected maximum total rewards, $v_i(n)$, for each state i. Being T the number of periods over a finite horizon and $v_i(n+1)$ the vector representing the expected maximum total rewards of the next epoch

$n+1$, the vector $v_i(n)$ represents the expected maximum total rewards earned from epoch n to T when in state i at epoch n considering all the actions:

$$v_i(n) = \max_k \left[q_i^k + \sum_{j=1}^{S} p_{i,j}^k v_i(n+1) \right], \quad \text{for } n = 0, 1, \cdots, T-1. \tag{2.14}$$

Then, vector d_i chooses for its value entries the action k that was used to compute each entry of $v_i(n)$.
Summarily, this procedure ends up choosing the action k for each state which provided more profit (or less cost) to the process[4].

In a finite horizon, this estimation of values can be completely determined as soon as one can say the final value of the process, i.e. the value of the ultimate reward earned with the process (which is a fixed value) if a process ends in final state i, this is called the salvage value of the process. Salvage values must be specified for all states and they are the entries of the last vector of expected maximum total rewards $v_i(T)$.

However, salvage values are the main problem when the horizon is infinite, i.e. there is no final period (T) to enable the value iteration procedure. Therefore, not only it must be assumed that the policy specified by the vector d_i must be stationary over an infinite planning horizon, which means that the policy will always specify the same decision in a given state regardless the epoch, but also it has to be found some extra conditions to enable/finalize the value iteration procedure.

There are four computational procedures that are used to solve an MDP problem over an infinite planning horizon: i) exhaustive enumeration, ii) value iteration, iii) policy iteration and iv) linear programming.

Exhaustive enumeration is computationally prohibitive unless the problem is extremely small. Value iteration requires less arithmetic operations than these alternative procedures, though it may never satisfy a given stopping condition. Policy iteration maximizes the gain or the average gain/reward per period. Finally, linear programming is formulated with the support of computer software packages, which are capable of solving both linear problems. Note that, though these are all approximated attempts for ideal perpetual models to solve MDPs, this field of investigation is still active nowadays with new optimized models and algorithms coming out.

Since we are dealing with economic values and balances, the last parameter to be considered for the MDPs is the discount factor, $\gamma \in [0, 1]$, which represents the difference in importance between future rewards and present rewards, it can be related with a discount rate (r)[5] and it is used to obtain the expected present values.

Hence, this new factor will also take part in the calculus of the expected total values, being now the expected total discounted value rewards. For a finite horizon, the discount factor γ is applied in the following way:

$$v_i(n) = \max_k \left[q_i^k + \gamma \sum_{j=1}^{S} p_{i,j}^k v_i(n+1) \right], \tag{2.16}$$
$$\text{for } n = 0, 1, \cdots, T-1.$$

To summarize the ideas, a decision problem within the MDP framework resides in a 5-tuple (s, a, p, q, γ):

- s is a certain state;
- a is a certain action available from state s;
- p is the probability that action a in state s will lead to the next state;
- q is the immediate reward received after transitioning from state s to the next one;
- γ is the discount factor which represents the difference in importance between present and future rewards.

The MDP approach is usually chosen for being the most attractive, fringe and advantageous for planning inspection/monitoring and maintenance based on stochastic models. Although MDPs can provide more versatile and non-problematic global optimum policies, these models share the limitation of perfect observations and a fully complete access to them in the timeline. Nevertheless, it happens that, especially in field of infrastructure management, these are unrealistic assumptions, and there is some measurement error associated with inspection activities.

In fact, all the non-destructive inspection techniques currently available can only approximately evaluate the real state of the structures, having some uncertainties in their measurements. Moreover, exact periodic inspections are not always possible, sometimes the structure must be in service during the epoch n, specified by the Markov chain. Due to the computational impossibility of developing more complex and exhaustive methods, to overcome these obstacles, other decision processes were developed based on quantities adequate enough for planning under uncertainty, instead of the information vectors withdrawn from the inspections (Papakonstantinou and Shinozuka, 2014 a). There are some extensions of the traditional MDP, which are useful to integrate the measurement error associated with inspection activities (see Madanat (1993) and Madanat and Ben-Akiva (1994) for further details). Other models, such as the latent Markov Decision Process or the Partially Observable

[4] A finite horizon practical example for this iterative method is shown in section 5.1.2.2.2 of the book *Markov chains and decision processes for engineers and managers*, Theodore J. Sheskin (2016).

[5] For example, $\gamma = \frac{1}{1+r}$.

Markov Decision Process (POMDP), provide flexible decision making frameworks specially for partially observable environments. These two extensions are outside the scope of the present thesis.

Chapter 3: Inspection, degradation and maintenance of railway wheelsets

Along this chapter, the main principles of the inspection, degradation and maintenance of railway wheelsets will be explained. Section 3.1 starts with a brief remark on the triad "inspection, degradation and maintenance". Section 3.2 starts by discussing the associated inspection activities, then section 3.3 looks at the degradation of railway wheelsets (namely wear and damage) and finally, section 3.4 explores maintenance actions.

3.1 – The triad "inspection, degradation and maintenance"

A former president of the UIC once said, *"The railway will be the 21st Century's preferred mode of transport – if it can survive the 20th Century."* (Nash et al. 2009). In the last 150 years, railway systems have been one of the top chosen means of transport for people, goods and commodities.

This choice has been sustained mostly in a reliable vehicle-wheel-rail system, which translates in high safety levels for the components' material, shape, dimensions, construction, geographic itineraries, rail trajectories, damping dynamics and accommodation of loads. From an engineer's point of view, these technical rules must be guaranteed with rigorous and recurrent check-ups of these parameters to ensure an acceptable interaction of vehicle and track - inspection (Iwnicki 2006).

The vehicle's system undergoes faults and failures with time, usage and ageing of materials. All the components involved in railway dynamics are subject to complex and concentrated forces due to the high speeds and loads associated with the locomotion. When the vehicle has travelled a certain distance, external factors such as number of trips, distance travelled per day, hot or cold climate conditions, mountainous, dusty or iced roads, heavy stop-and-go cruising also help increasing the natural phenomenon of degradation of the wheelsets (Sharma 2016).

To guarantee the longevity of railway structures, it is needed to correct, improve or replace the affected components, always with the vision of maximum durability, efficiency and economy in mind. When every factor, which could influence positively the running of a process, is properly managed, by performing routine actions for keeping devices, equipment, machinery, infrastructures or even supporting utilities in working order by also preventing trouble from arising, these all sets of actions are called maintenance. They can be condition-based, corrective, planned, predictive or preventive operations (Lambert 2016).

These three factors – inspection, degradation and maintenance - are symbiotically interconnected in every health condition monitoring engineering system. Inspection mainly detects safety and/or condition parameters/indicators that are not being met, due to, among others, degradation. To put back on track, i.e. recover, the safety/quality/condition levels, it is necessary to act according to maintenance policies that are defined with the help of precise inspections in the structure.

In the next sections, this engineering triad (inspection, degradation and maintenance) will be explored for the railway wheelset component.

3.2 – Inspection

Inspection activities are a mean to detect either functional or safety failures. There are several parameters in the wheelsets shape, size and weight, which need frequent and rigorous inspections. In agreement with the UIC 510 – 2 OR standard (2004), this procedure is based on existing rules and practices such as (UIC 2004):

- Limiting measurements for wheel manufacture and re-profiling;
- Limiting measures for wheelsets operating;
- Permissible weights per axle;
- Determining track characteristics;
- Specifying the wheelsets' material properties and treatment;
- Limiting operating temperatures.

As referred in a British Standard (BS) , due to numerous international regulations, new railway vehicles have to be tested and homologated before they are put in service, the same happens when the operating conditions have to be extended. With the significant increase of international traffic, the standardization of existing regulations is required and in some cases additional rules have to be added. Therefore, there have been settled specific zones and measure relations for both axle and wheels of a wheelset (BS 2005).

The main focus on the wheel inspection would be at analysing its wheel-profile specifications and the associated maintenance actions are going to be explored further in section 3.4.

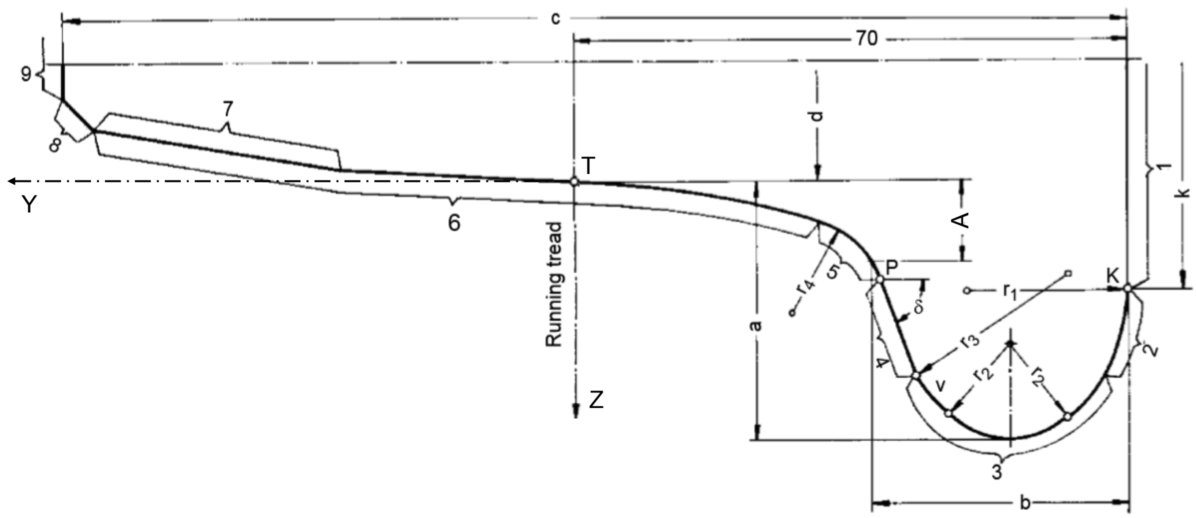


Figure 3.1: Wheel running profile specifications, adapted from UIC (2004).

Table 3.1: Descriptions regarding the wheel running profile.

Reference number	Description the of wheel-profile zone	Reference letter	Description of the wheel-profile measure
1	Internal surface of the rim	a	Flange height
2	Internal surface of the flange	b	Flange thickness
3	Top of flange	c	Width rim
4	External surface of flange	d	Diameter of running tread
5	Running profile fillet	r_1, r_2, r_3	Radii of rounded end of flange
6	Running surface	r_4	Radius of running profile fillet
7	Slope of external section of the running surface	δ	Angle of external surface of flange
8	External bevel of running profile		
9	External surface of the rim		

In every railway wheel, there is a tread datum position (point T in Figure 3.1), which is located on the running surface 70 mm from the internal surface of the rim, and from which the location of any given point on the wheel is relative to that point and measured from it.

Particularly due to high speed traffic and technological progress, not only an update of existing regulations is needed, but also more accurate measurement methods must be introduced. The more precise the inspection/measurement of a wheelset is, the better are the data results withdrawn from that inspection and better decisions would be taken in principle. In that sense, being able to better classify how far is the wheelset condition/state from the safety limits allows to better predict the next period of inspection (Andrade and Stow 2016).

Regarding the measurement of wheels, it has traditionally been made by the use of a gauge device (Figure 3.2). Its positioning on the wheel is made by a roller on the tread datum position, situated 70 mm from two circular magnetos positioned on the internal surface of the rim, responsible for the device fixation. This device just measures three of the called wheel functional references: the flange height, the flange thickness and the flange angle of external surface. These measures are withdrawn and read in a similar way of a Vernier calliper with the device tips touching the wheel surface.

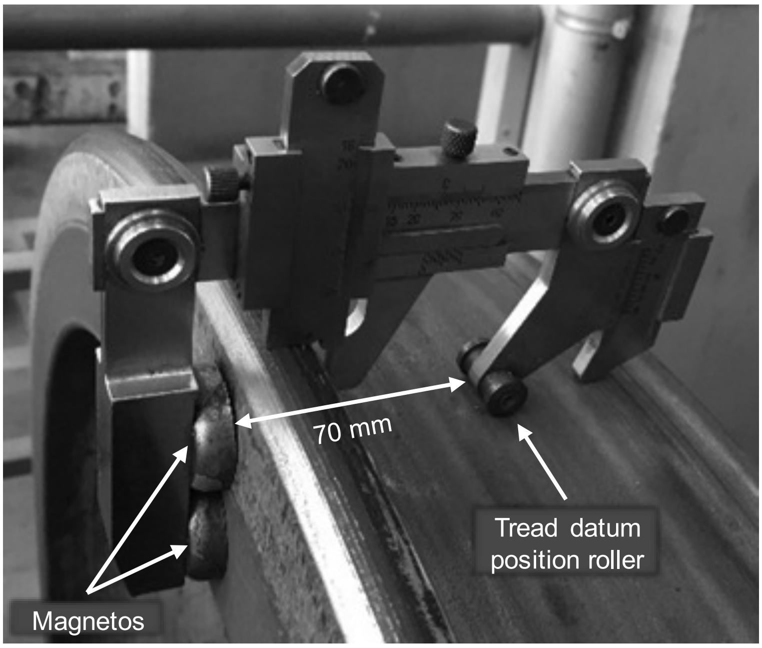


Figure 3.2: Gauge device.

The maximum precision of a gauge device (as pictured in Figure 3.2) is approximately ± 0,05 mm. However, gauge devices are prone to human errors, such as bad positioning of the instrument relative to the wheel and reading errors using the nonius. Therefore, for the same wheelset inspection different operators might get measurements 4 mm apart.

There is also a fringe technique using a laser equipment (Figure 1.5b and Figure 3.3), being an alternative to error-prone callipers. This is a contact-free technology that evaluates all the wheel profile lengths. The operator approaches to the wheel surface a scan device that emits three laser lines responsible for processing an intelligent image of the wheel shape on a portable screen. Once the wheel body contour is totally defined on the screen, the information system indicates if the wheel values are within or beyond the tolerance limits.

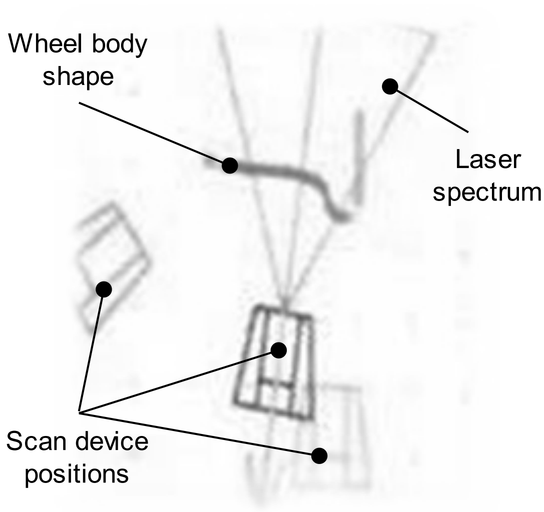
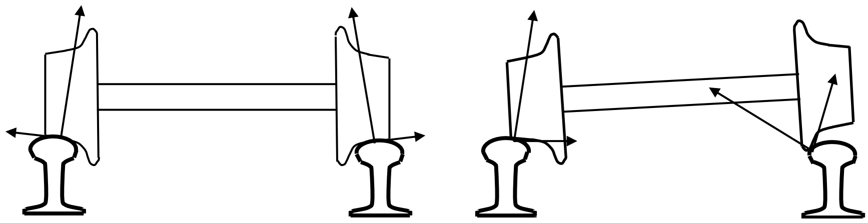


Figure 3.3: Image observed in the informatic program of the laser equipment when scanning the wheel body.

The precision of this equipment is approximately ± 0,01 mm and it is not subjected to human errors, being a much more reliable and accurate inspection tool than the previous traditional one. Concerning the damage occurrence on the wheelsets surfaces, the detection methods can be manual, visual or automatic (including trackside detection equipment). The most used non-destructive testing (NDT) methods on wheelset inspection are: visual testing (VT), magnetic particle testing (MT) and ultrasonic testing (UT). The biggest share of damage occurrence is detected by VT, only then MT takes place. UT is only used as a last resort, being that many companies do not possess this equipment for being expensive.

3.3 – Degradation

The wheel-rail contact is a complex imperfect system composed by parts of rolling interaction and parts of sliding interaction. In each point of the wheel, there are normal and lateral tangential forces applied in the rolling surface (Figure 3.4).



(a) (b)

Figure 3.4: Normal and lateral tangential forces acting on wheelsets: (a) in central position and (b) in laterally displaced position, illustrating the gravitational stiffness effect.

The wheelset guarantees the vehicle guidance on the track, due to flanges[6] and the conic profile of the wheels (Magalhães 2013). The rolling mechanism is most of times limited by the transverse play, with partially sliding surfaces without protection against dust, rain, sand or ballast stones (Figure 3.5).

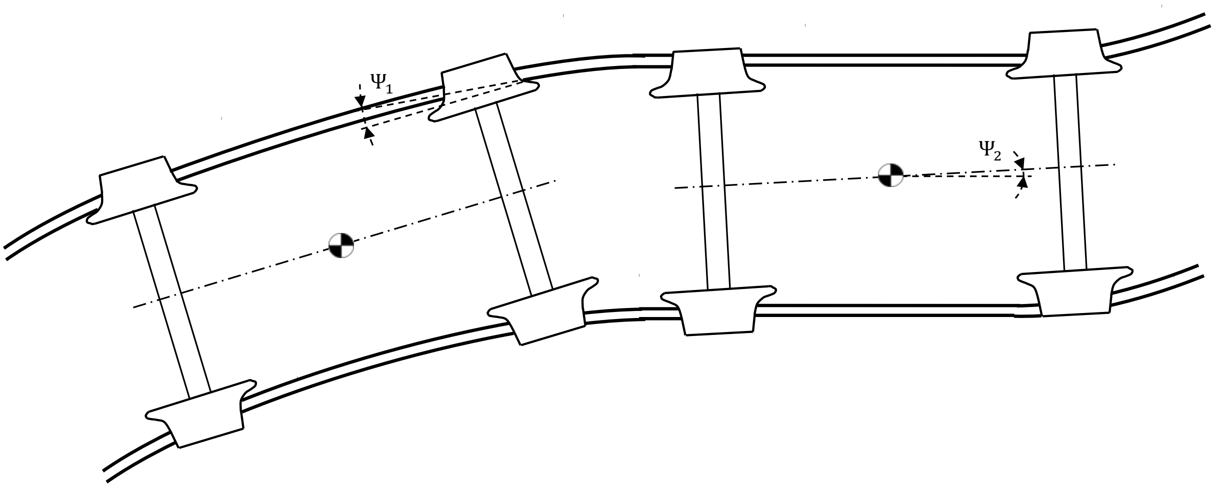
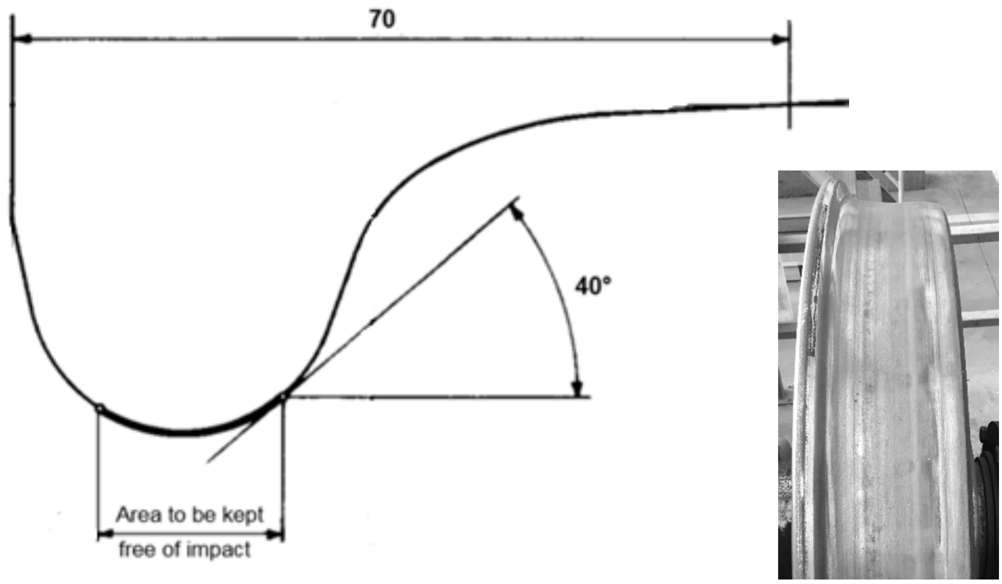


Figure 3.5: Rail-wheel transverse sliding contact zones with different displacement angles (Ψ_1, Ψ_2) for different rail trajectories.

Both rolling and sliding occur in the contact zone (Figure 3.6), especially in curves, there can be a large sliding component on the contact as well as larger lateral forces.

[6] The aim of the flange is to keep the vehicle in the track when contact with the conic part of the wheel is exceeded. The ratio between the lateral and normal forces should not exceed values between 0,8 and 1,2 so that the risk of derailment can be neglected.



(a) (b)

Figure 3.6: Contacting zone: (a) area free of impact for a standard wheel (on its right, the contacting zone), adapted from UIC (2004), (b) wheel rolling surface with some mileage.

These sliding frictions, as well as the climate external factors are responsible for changes in shape in some zones in the rolling tread and/or other damage defects that may occur during the wheel's life cycle.

In Figure 3.7, the changes in shape of the rail and wheel caused by the contact wearing are shown (Iwnicki 2006).

Figure 3.7: Form change of wheel and rail from the Stockholm test case (Iwnicki, 2006).

Apart from the changes in shape, it can also occur damages in the rolling surface . Nowadays, this is a very sensitive issue in the railway industry mainly due to the development of new high-speed railway lines which need a progressive process of adaptation between the new vehicles with higher speeds and the upgrade of the existing railway infrastructures. The increasing demands on railway transportation require improvements of the network capacity, which can be achieved either by increasing the speed of the traffic or by increasing the axle loads. Meanwhile, these facts result in several types of damage in the wheelsets than before, which is an aspect of the wheelset degradation that still needs to be given greater attention and improvement because these problems can significantly shorten a wheel's life (Pombo et al. 2011).

According to the BS (2012), several types of damage are verified, such as pitting, rolling contact fatigue (RCF), hard spots, rollover, cavities, wheel flats, build up, indentations, cracks or spalling. As reported by Andrade and Stow (2016), the three more significant types of damage are: wheel flats, cavities and RCF.

Figure 3.8: The three more significant types of damage: (a) wheel flats, (b) cavities, (c) RCF (BS 2012).

3.4 – Maintenance

As the wheelset wears or a damage occurs, certain safety levels have to be re-ensured, thus, maintenance actions have to be periodically and strategically applied. The sequence, the exact timing/period and types of operations are defined by the maintenance plan. Limit values adopted are on the basis of service experience, being established limits for in-service and off-vehicle wheelsets. The maintenance plan shall specify measures to be implemented, which actions should be performed to meet the requirements and mandatory operations listed in the standards, the corrective actions necessary for dealing with defects and the time intervals between maintenances. With service experience, it is then possible to add/remove rules to further enhance the maintenance in a dynamic feedback iterative process (Figure 3.9).

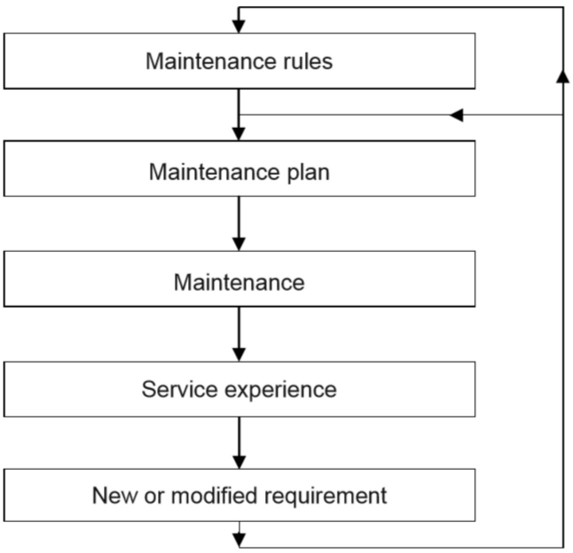


Figure 3.9: Self-improving feedback iterative process of maintenance (BS 2012).

Another maintenance specification is to ensure traceability of every wheelset, thus wheelsets should have owner’s mark on the wheel and external identification (tag, metal plate or collar) on the axle. All the identification marks companied its corresponding maintenance life history shall be described in a document to support the management of the wheelsets during their service lives.

Finally, the maintenance plan is made by qualified people and approved by certified entities as well as the staff carrying out NDT and welding operations (BS 2012).

Therefore, these several aspects previously explained can be organized as represented in Figure 3.10.

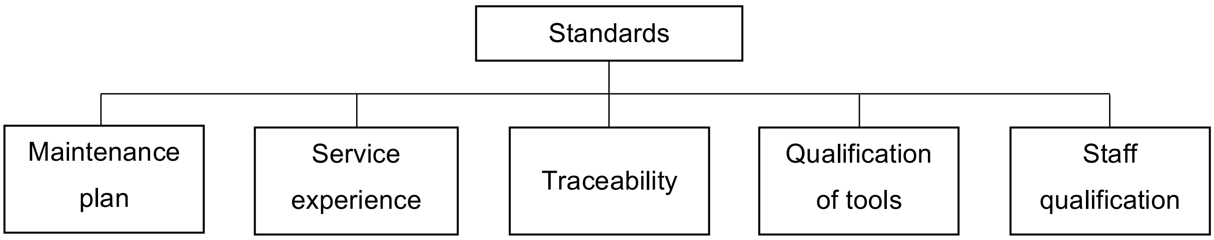


Figure 3.10: General maintenance organization.

Besides being a simpler component, the axle has far less level of adverse exposure for damage than the wheels, as it barely changes its shape during service-time, and hence, it does not need as many periodic inspections as the wheels. Due to higher complexity of the evolution of the wheelset shape in service and the occurrence of damage, there is more room for improvement in the maintenance of the wheels rather than in the axle, thus, the maintenance of the former wheelset component will be the main focus henceforth.

To define a maintenance plan for a wheel, several aspects must be bear in mind (Figure 3.11) related with the tribology of the wheel-rail contact and economic considerations (Iwnicki 2006).

Figure 3.11: Systems aprroach to wheel-rail interface reseach and development. Nothing can really be treated in isolation (Iwnicki 2006).

A comprehensive degradation model for a wheel would assess the evolution of three geometrical variables throughout the life cycle of the wheel:

- The wheel diameter (D);
- The flange height (F_h);
- The flange thickness (F_t).

The diameter (letter d in Figure 3.1) is measured from one tread datum point to the opposite one in a wheel section, the flange height (letter a in Figure 3.1) is the distance between the top of flange and the tread datum position, finally the flange thickness (letter b in Figure 3.1) is the distance between the internal surface of rim and a surface end point situated A mm from the tread datum position. This A distance varies depending on the standard adopted for the wheel profile, having different values according to the country (Andrade and Stow 2016).

The control and the way of acting on these three parameters is going to reduce the operation and maintenance costs, by increasing the life cycle of the wheelset and consequently both vehicles and tracks. When the worn state of the wheel-profiles reaches the limit value defined by the standards, the wheels have to be re-profiled. Railway wheels can only be re-profiled several times and the wheelset substitution is a very expensive maintenance procedure. Furthermore, we have also to consider damage problems, which can significantly shorten a wheel's life as they often require a large reduction in the wheel diameter in order to remove the damaged material (Pombo et al. 2011).

International standards provide limits for the maximum flange height and minimum flange thickness. If a wheel arrives to an inspection bench infringing one or both these limits, then wheel turning is needed to be restored to reasonable values within the regulatory parameters. The same happens if the wheel exhibits damage, then it needs re-profiling beyond the damaged zones, removing the imperfections with the turning process. The differences between the initial flange values and the ones after turning are defined as ΔF_{h_T} and ΔF_{t_T}, respectively for the flange height and flange thickness variation. Making the decision to turn a wheel imposes a loss in the wheel diameter (ΔD_T). Once the wheel reaches its scrap diameter (D_{Scrap}), the vehicle must be removed from service, and the wheelset replaced.

On the other hand, on the wear trajectory, the corresponding geometrical variations can be quantified as (Andrade and Stow 2016):

- the change in the flange height (ΔF_h);
- the change in the flange thickness (ΔF_t);
- the change in diameter (ΔD).

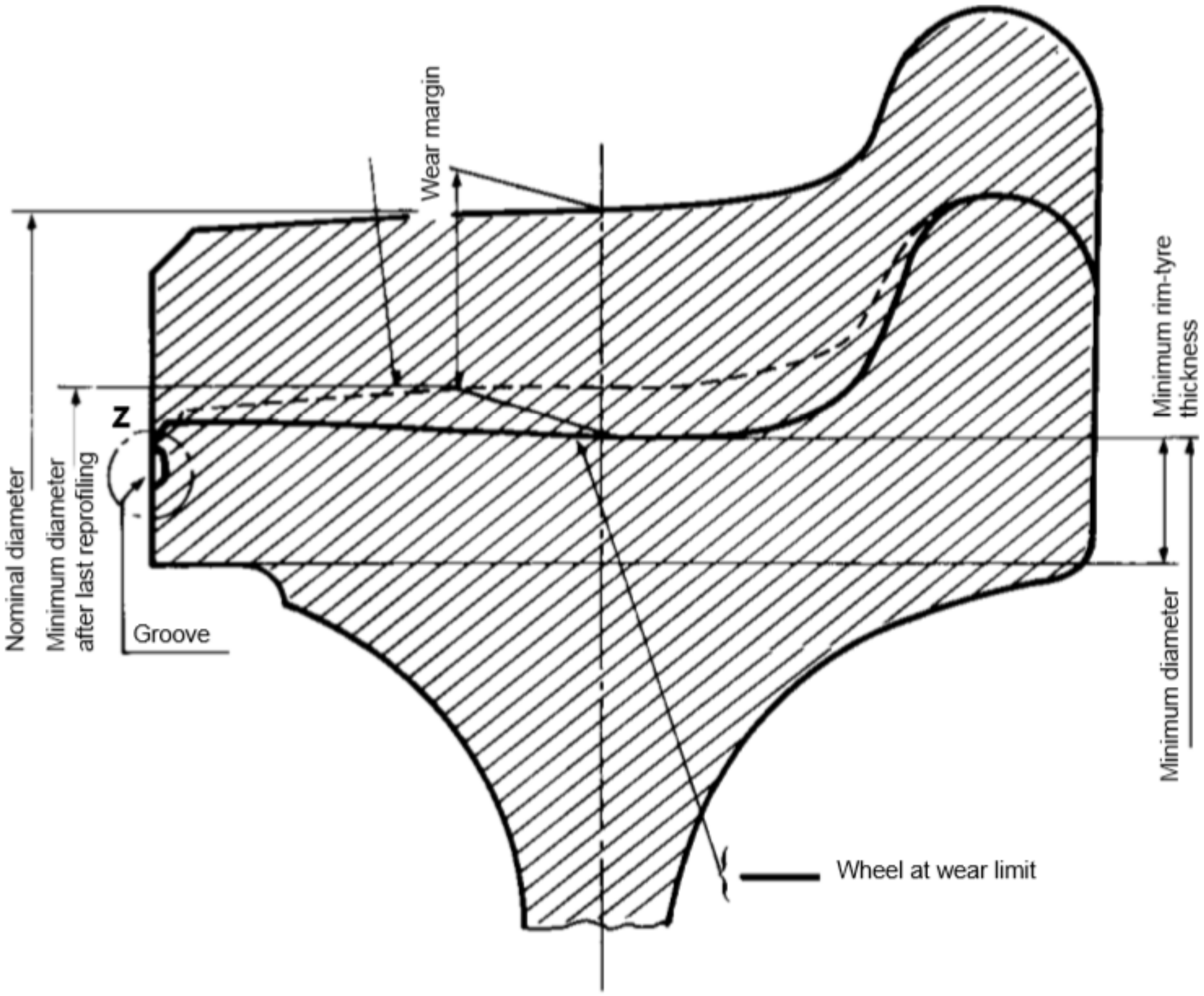


Figure 3.12: Wear margin for a profile with minimal permissible diameter following last re-profiling and scrap diameter reference groove (UIC 2004).

Chapter 4: A practical example

In this chapter, the contents covered in the previous chapters are gathered and applied in a practical example of wheelset maintenance. Its final objective is to determine an optimal strategy for the maintenance of railway wheelset. Section 4.1 provides the estimation of the Markov transition matrices (MTMs) for each possible action and Section 4.2 discusses the reward/cost functions. Finally, Section 4.3 provides the optimal maintenance policy.

Not that, as it was not possible to get in useful time the wheelset condition data from a Portuguese train operating company, a past sample (Andrade and Stow 2016, 2017a, 2017b) was used to estimate MTMs. The analysed database compiled wheel data from December 2006 up to July 2012 (i.e. a 7-year interval) from a single fleet of train (i.e. it only contains trains of one type or class). Each unit has three vehicles and each vehicle has eight wheels (i.e. four wheelsets). Further details on this sample can be found in those references (Andrade and Stow 2016, 2017a, 2017b).

4.1 – Estimation of Markov transition matrices

For a Markov chain with a finite but large state space, its decomposition tends to follow the most attractive approach. Having said that, the underlying problem is divided into sub-problems that can be solved independently. Considering a homogenous Markov chain, the transitions matrix is decomposable into several sub-transition matrixes – in a diagonal block form. The real world is not ideal, and consequently, rather than complete decomposability, one encounter nearly completely decomposable cases (Yin and Zhang 2006).

Points in state space that are close to each other will share the same action, and fortunately there are some mitigating constraints that can be exploited: first, only a relatively small part of the state space will actually be visited during any normal wheelset deterioration process; second, the range of actions at any specific point are restricted (Young et al. 2013).

4.1.1 – Data analyses from past sample

The evolution of Markov models/chains are strongly corroborated by data analysis of past samples. To apply the Markov properties to this case study, a brief exploratory analysis guarantees linear independence between the main variables that model the evolution of the wheel shape.

By observing Figure 4.1, it is possible to see that there is not a linear correlation[7] between the variation in diameter for each wheel due to wear (ΔD) and the size (state) of the wheel (D), for the wheels analysed.

Therefore, it cannot be rejected that the wear in a wheel and its change in diameter is statistically independent of its initial state (at least linearly).

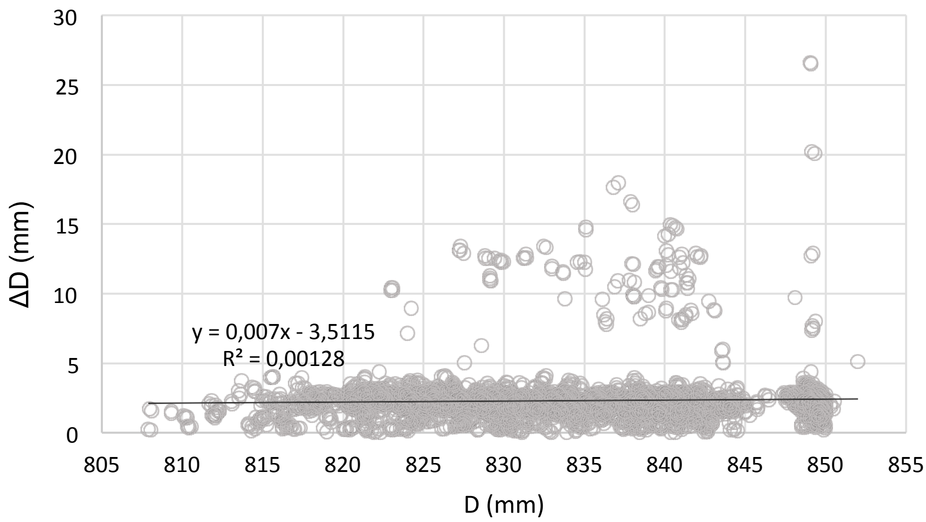


Figure 4.1: Change in the tread diameter due to wear (ΔD) for different initial diameters (D).

The diameter of the wheels must respect the standard limit sizes and it is a key indicator of the stage in the life-cycle in which a given wheel is at a certain period. Consequently, it is the main indicator that will be studied and modelled in the following estimation of the MTM. In the sample being analysed, the wheelset diameter (which must be very similar for the right and left side positions) varies from an initial diameter of around 850 mm to a scrap diameter of around 790 mm (below which, it must be renewed).

4.1.2 – A simple approach to estimate transitions matrices

For this practical example, the state space was first divided in 60 states, depending on the values of the wheel diameter between 850 mm and 790 mm and grouped with differences of 1 mm:

$$s = \left\{ \begin{array}{cccc} s_1 = 1 & s_2 = 2 & \cdots & s_{60} = 60 \\ \downarrow & \downarrow & & \downarrow \\ D \in [850, 849[& D \in [849, 848[& & D \in [791, 790[\end{array} \right\}, \quad D \text{ in mm.} \tag{4.1}$$

Nevertheless, as two wheelsets can have the same value of diameter, but be in different states due to the occurrence of damage, another subset of 60 states was included. Therefore, the number of states doubles:

$$s = \left\{ \underbrace{\begin{array}{ccc} s_1 = 1 & \cdots & s_{60} = 60 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{array}}_{Without\ damage} \underbrace{\begin{array}{ccc} s_{61} = 61 & \cdots & s_{120} = 120 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{array}}_{With\ damage} \right\}, \quad D \text{ in mm.} \tag{4.2}$$

Moreover, as shown later on, the mileage since last turning (or renewal)[8] is an important variable that explains the occurrence of damage, thus, it is an information that must be taken into account in the definition of the set of possible states. Therefore, it was decided to divide the state space without damage in more 26 subsets depending on the mileage since last turning (mst), in intervals of 10 thousand miles, from 0 miles up to 250 thousand miles:

$$s = \left\{ \begin{array}{l} \underbrace{\begin{array}{ccc} s_1 = 1 & \cdots & s_{60} = 60 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{array}}_{\substack{Without\ damage \\ and\ mst=0\ k\ miles}} \underbrace{\begin{array}{ccc} s_{61} = 61 & \cdots & s_{120} = 120 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{array}}_{\substack{With\ damage \\ and\ mst=10\ k\ miles}} \\ \underbrace{\begin{array}{ccc} s_{121} = 121 & \cdots & s_{180} = 180 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{array}}_{\substack{Without\ damage \\ and\ mst=20\ k\ miles}} \cdots \underbrace{\begin{array}{ccc} s_{1501} = 1501 & \cdots & s_{1560} = 1560 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{array}}_{\substack{Without\ damage \\ and\ mst=250\ k\ miles}} \end{array} \right. \tag{4.3}$$

---

[7] $R^2 = 0{,}0013$

[8] Hereinafter, whenever it is mentioned the mileage since last turning, the renewal situation is always englobed.

$$\underbrace{\begin{matrix} s_{1501} = 1561 & \cdots & s_{1560} = 1620 \\ \downarrow & & \downarrow \\ D \in [850, 849[& & D \in [791, 790[\end{matrix}}_{With\ damage} \quad \} ,$$

D in mm.

Note that the 60 states with damage are kept at the end of the state space, but without the extension depending on the mileage since last turning, because the transitions from damaged states to non-damaged states are compulsory, since that once the damage is detected it must be removed. Although the number of states described above might seem at first very large, it simplifies the estimation matrices, as it will be clear later on.

A sub-transition matrix will have to be defined for each possible action. Note, that certain simplifications were adopted as the defined states do not control other parameters such as the flange thickness or height and angle dimensions.

After a wheelset inspection, three actions ($a = 1, 2, 3$) can be performed:

- "Do nothing" ($a = 1$): the wheelset is ok and it goes back to service in the same state;
- "Renewal" ($a = 2$): the corrective or preventive maintenance actions would need to go beyond the scrap diameter, and so the wheel must be replaced by a new one;
- "Turning" ($a = 3$): the wheelset goes to a turning lathe for its shape being replaced to values within the standards and it suffers a reduction/loss in its diameter.

– "Do nothing" action ($a = 1$):

Relatively to the "do nothing" action, since in the transitions analysed, derived solely from states of wear of the wheels, the probability of an increase of diameter size is zero, hence, a transition to a state with higher diameter is considered impossible.

On the other hand, it was verified by the data analysis that the transitions to next states are limited, which means that a transition from a state to another one with a great loss in the diameter due to wear, for example, is very unlikely to happen. Therefore, from what concerns transitions from one state to the other ones, the probabilities are composed by zeros to states before the current one and zeros for the states very unlikely and not found by the data to happen.

As a first approach of MTM estimation for the wear situation, damage is not considered (i.e., Equation (4.1) would be the state space). The independency of the variation in diameter for each state of the wheel, assumed in the subsection 4.1.1, leads to the conclusion that the probability of a state transition can be extrapolated for every single state in the pattern, regardless of the value of the diameter (i.e. the state). Letting s' be the next state, the values different from zero for every transition are obtained as:

$$p_{i,j}(s'|s,a) = p_{i+1,j+1}(s'|s,a). \tag{4.4}$$

To respect the state space restriction of the scrap diameter, i.e. a worn state cannot evolve to a state that goes beyond it, the remaining probabilities (that would indicate a transition to a state with a diameter smaller than the scrap diameter) are going to be summed up (in the last column of the MTM) to represent the joint probability of the wheelset remaining in the last final diameter state.

Therefore, in a matrix form, the sub-transition matrix for the wear situation without damage would adopt the following aspect:

$$P_{Wnd} = \left[p_{i,j}^{1}\right] = \begin{bmatrix} p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^1 & p_{1,2}^1 & p_{1,3}^1 & \cdots & p_{1,j}^1 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^1 & p_{1,2}^1 & \cdots & p_{1,j-1}^1 + p_{1,j}^1 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^1 & \cdots & p_{1,j-2}^1 + p_{1,j-1}^1 + p_{1,j}^1 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \sum_j p_{1,j}^1 = 1 \end{bmatrix}. \tag{4.5}$$

For this dissertation it is going to be assumed that it is only possible for the wheelset to transit to the state of diameter immediately below with a probability of θ, as described in Figure 4.2.

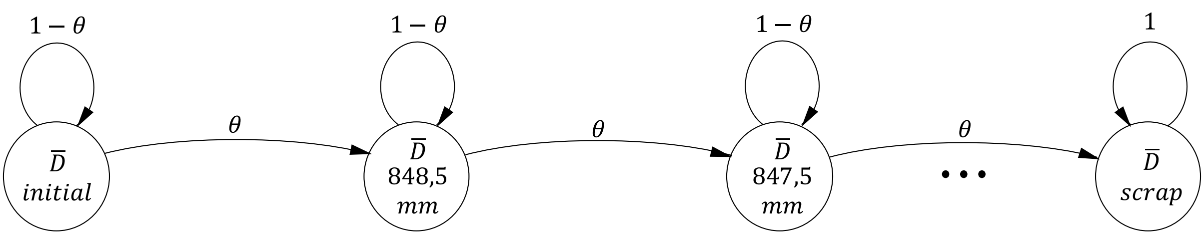


Figure 4.2: Transitions between states without damage using a probability value θ for the “do nothing” action.

Hereupon, the sub-transition matrix of Equation (4.5) would adopt now the following simpler form:

$$P_{Wnd}\left[p_{i,j}^{1}\right] = \begin{bmatrix} 1-\theta & \theta & 0 & \cdots & 0 \\ 0 & 1-\theta & \theta & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & 1-\theta & \theta \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}. \tag{4.6}$$

In what concerns the tread change diameter due to wear, it is possible to predict a mean value ($\overline{\Delta D}$) for this wearing evolution using a Markovian approach, with time intervals of 10 thousand miles and a MTM.
Being

$$\Delta D = [0 \quad 1 \quad 2 \quad 3 \quad \cdots \quad 59]^T \; (mm), \tag{4.7}$$

every possible values for changes in diameter[9] and

$$X_0 = [P(\Delta D = 0) \quad P(\Delta D = 1) \quad \cdots \quad P(\Delta D = 59)] = [1 \quad 0 \quad \cdots \quad 0] \tag{4.8}$$

the initial state of the wear process (when the wheel has no mileage since last turning (mst), there is also no change in its diameter), it is possible to predict every future state of deterioration in intervals of 10 thousand miles according with the development of Equation (2.7), and then, according to Equation (2.12), derive a scalar mean value $\overline{\Delta D}_{(n)}$ for each mileage since turning value interval :

$$\overline{\Delta D}_{(n)} = X_0 P^n \Delta D. \tag{4.9}$$

By choosing different values for the variable θ, an average loss of diameter due to wear can be estimated using Equation (4.8) and compared with real data from Andrade and Stow (2016, 2017a, 2017b) sample as it is shown in Figure 4.3.

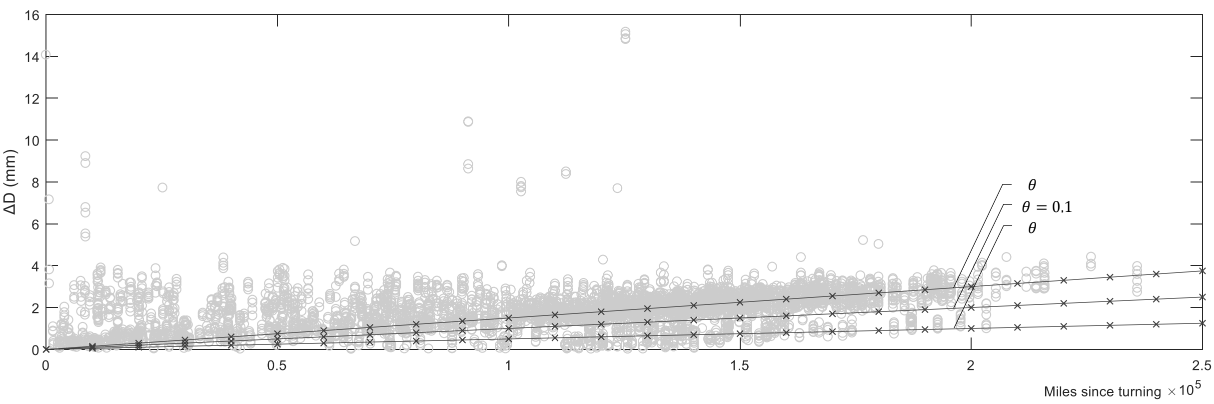


Figure 4.3: Change in the tread diameter due to wear (ΔD) for wheelsets without damage with mileage since turning, applying the first MTM approach.

In the case of Figure 4.3, choosing 0,15 as the probability value for θ would be the best aprroach comparing to the real data.

[9] Here were used the states representative values (i.e. diameter categories mean values) from $\overline{D}_{initial} = 849{,}5 \; mm$ up to $\overline{D}_{scrap} = 790{,}5 \; mm$.

Now, by expanding the state space to the one in Equation (4.3). The non-zero probability transition values of the MTM, if the "do nothing" action is chosen ($a = 1$), and not considering the existence of damage, would be the following:

$$\begin{cases} p_{i,i+60} = 1-\theta & , \quad i = j + 60k \\ p_{i,i+61} = \theta & , \quad i = j + 60k \\ p_{i,i+60} = 1 & , \quad i = 60(k+1) \\ p_{i,i} = 1 & , \quad i = 1501, \quad 1502, \quad \cdots \quad 1560 \end{cases} \quad , \quad j \in \{1,2,\cdots,59\} \quad , \quad k \in \{0,1,\cdots,24\}. \tag{4.10}$$

Note that the last condition of Equation (4.10) has to be inserted because of the mst restriction (for better understanding, see Appendix A1).

At the moment, no transition probabilities to states with damage have been defined and the probabilities of occurring damage should be considered now. The following modelling approach considers only the possibility of a wheelset acquiring damage without change in its diameter, as shown in Figure 4.4.

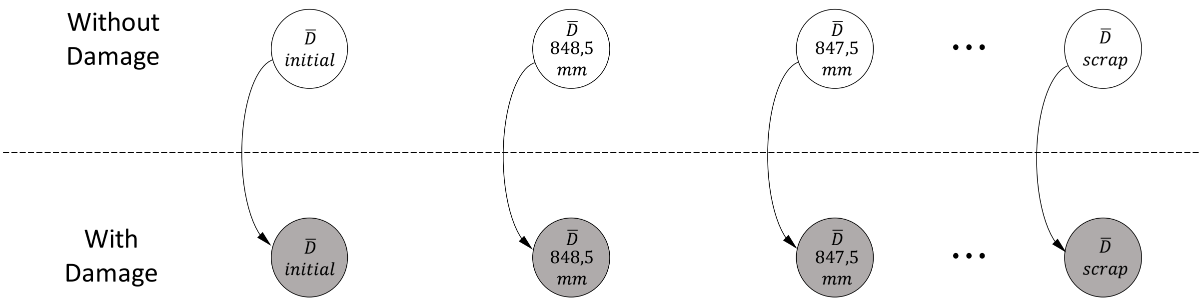


Figure 4.4: Considered transition probabilities to states with damage.

In Andrade and Stow (2016), the authors used a logistic regression to estimate the probability of occurring damage, given a certain number of explaining variables, namely: i) mileage since last turning and ii) the wheelset diameter. They used the following expression:

$$p(damage) = \frac{1}{1 + e^{-(\sum_i \beta_i X_{ij})}} \tag{4.11}$$

where j, β_i are the slope parameters associated with each covariate i, and X_{ij} are the values for each covariate i and wheelset j. Figure 4.5 estimates the probabilities of occurring damage (here assumed solely as Rolling Contact Fatigue), for the damaged transitions considered in Figure 4.4, with the evolution of the mileage since last turning.

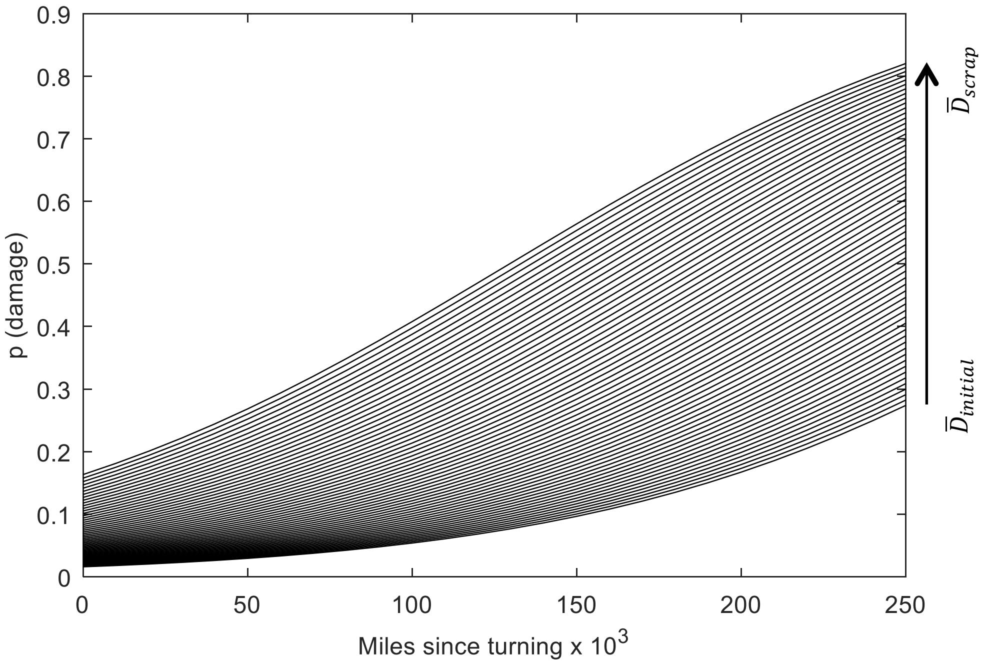


Figure 4.5: Probability of damage with mileage since last turning.

A sub-transition matrix for the probability values of Figure 4.5, which transitions are schematically represented in Figure 4.4, can be represented from the different 1560 states without damage to the different 60 states with damage in the following way:

$$P_D = \left[p^{1}_{i,j}\right] = \overset{\bar{D}\ \underset{damaged}{initial} \longrightarrow \bar{D}\ \underset{damaged}{scrap}}{\left[\begin{array}{cccc} p(damage) & 0 & \cdots & 0 \\ 0 & p(damage) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & p(damage) \\ \hline p(damage) & 0 & \cdots & 0 \\ 0 & p(damage) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & p(damage) \\ \hline p(damage) & 0 & \cdots & 0 \\ 0 & p(damage) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & p(damage) \\ \hline p(damage) & 0 & \cdots & 0 \\ 0 & p(damage) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & p(damage) \end{array}\right]} . \begin{array}{l} \bar{D}_{initial} \downarrow \bar{D}_{scrap} \Big\} \; mst\ 0\ miles \\ \bar{D}_{initial} \downarrow \bar{D}_{scrap} \Big\} \; mst\ 10k\ miles \\ \bar{D}_{initial} \downarrow \bar{D}_{scrap} \Big\} \; \vdots \\ \bar{D}_{initial} \downarrow \bar{D}_{scrap} \Big\} \; mst\ 250k\ miles \end{array} \qquad (4.12)$$

The addition of damage to the problem causes some particular alterations in the transition probability values for the sub-transition matrices between states without damage, as in the case of Equation

(4.6), since now these probability values also have to guarantee that damage has not occurred. For these cases, it is assumed the independence between wear and damage occurrence and the joint probability of two independent events factorizes into their marginal probabilities (Puterman 2014):

$$P(wear \cap damage) = P(wear) \cdot P(damage) \tag{4.13}$$

Therefore, the sub-transition matrix of Equation (4.6) when considering the damage occurrence becomes as follows:

$$P_W = \left[p_{i,j}^1\right] = \begin{bmatrix} (1-\theta)(1-p(damage)) & \theta(1-p(damage)) & 0 & \cdots & 0 \\ 0 & (1-\theta)(1-p(damage)) & \theta(1-p(damage)) & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & (1-\theta)(1-p(damage)) & \theta(1-p(damage)) \\ 0 & \cdots & \cdots & 0 & 1-p(damage) \end{bmatrix} \tag{4.14}$$

Adding now the effect of damage to the Equation (4.10), the non-zero probability transition values of the MTM if the “do nothing” action is chosen ($a = 1$) would be then the following:

$$\begin{cases} p_{i,i+60} = (1-\theta)(1-p(damage)) & , \quad i = j + 60k \\ p_{i,i+61} = \theta(1-p(damage)) & , \quad i = j + 60k \\ p_{i,i+60} = 1-p(damage) & , \quad i = 60(k+1) \\ p_{i,i} = 1-p(damage) & , \quad i = 1501, \quad 1502, \quad \cdots \quad 1560 \end{cases} \quad , \quad j \in \{1,2,\cdots,59\} \quad , \quad k \in \{0,1,\cdots,24\} \tag{4.15}$$

For example, assuming that a given wheelset is new, i.e. it is currently in state s_1 ($mst = 0$, $D \in [850, 849[$ and no damage), if action “do nothing” ($a = 1$) is chosen, then three transitions are possible:

i) Damage occurs with a probability value of $p(damage)$ according to Equation 4.11. It transits from s_1 to s_{1561}, i.e. it moves from a state without damage to a equivalent one but with damage.

ii) No damage occurs and the diameter is not significantly changed by the wear with a probability value of $(1-\theta)(1-p(damage))$. It transits from s_1 to s_{61}, i.e. it moves from a state with $mst = 0$ to a equivalent one but with $mst = 10k\ miles$.

iii) No damage occurs and the diameter is changed by the wear with a probability value of $\theta(1-p(damage)$. It transits from s_1 to s_{62}, i.e. it moves from a state with $D \in [850, 849[$ to an equivalent one but with $D \in [849, 848[$.

The cases for the remaining states can be seen in better detail consulting Appendix A1.

In a matrix form, the final MTM for the "do nothing" situation (P_1), composed by the sub-transition matrices of Equations (4.12) and (4.14) in a diagonal block form, is as follows:

$$P_1 = \left[p^1_{i,j}\right] = \begin{array}{c} \begin{array}{ccccccc} \leftarrow mst=0\ miles & \leftarrow mst=10\ miles & \leftarrow mst=20\ miles & \cdots & \leftarrow mst=240k\ miles & \leftarrow mst=250k\ miles & \leftarrow States\ with\ damage \end{array} \\ \begin{bmatrix} 0 & P_W^{(60\times60)} & 0 & 0 & 0 & 0 & \\ 0 & 0 & P_W^{(60\times60)} & 0 & 0 & 0 & \\ 0 & 0 & 0 & \ddots & 0 & 0 & P_D^{(1560\times60)} \\ 0 & 0 & 0 & 0 & P_W^{(60\times60)} & 0 & \\ 0 & 0 & 0 & 0 & 0 & I^{(60\times60)} & \\ 0 & 0 & 0 & 0 & 0 & 0 & I^{(60\times60)} \end{bmatrix}^{(1620\times1620)} \begin{array}{l} \leftarrow mst=0\ miles \\ \leftarrow mst=10\ miles \\ \vdots \\ \leftarrow mst=240k\ miles \\ \leftarrow mst=250k\ miles \\ \leftarrow States\ with\ damage \end{array} \end{array} \quad (4.16)$$

– "Renewal" action ($a = 2$):

Relatively to the renewal action, independently of the current state of the wheel (damaged or undamaged), it is certain that it goes to the initial state, as described in Figure 4.6.

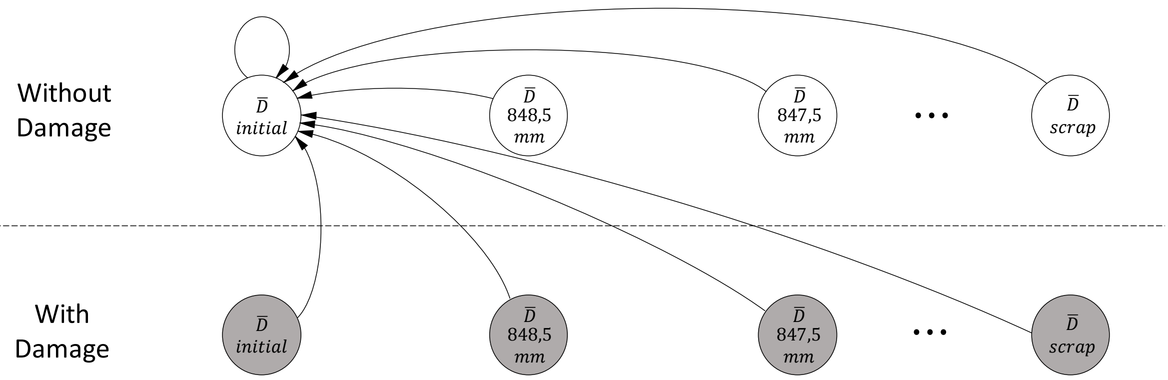


Figure 4.6: Transitions between states for the "renewal" action.

Therefore, the MTM for the "renewal" situation (P_2) is a 1620 by 1620 matrix (according to Equation (4.3) for the definition of the set of states) as follows:

$$P_2 = \left[p^2_{i,j}\right] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}^{(1620\times1620)} \quad (4.17)$$

The transitions between all the states considered at the final state space of Equation (4.3) and their probability values can be seen in Appendix A2.

– "Turning" action ($a = 3$):

For the “turning” action, a distinct loss in the diameter due to turning (re-profiling of the wheel) is achieved if damage has occurred or not. In case it has occurred, the diameter loss tends to be significantly larger on average and with an higher dispersion as depicted in Figure 4.7.

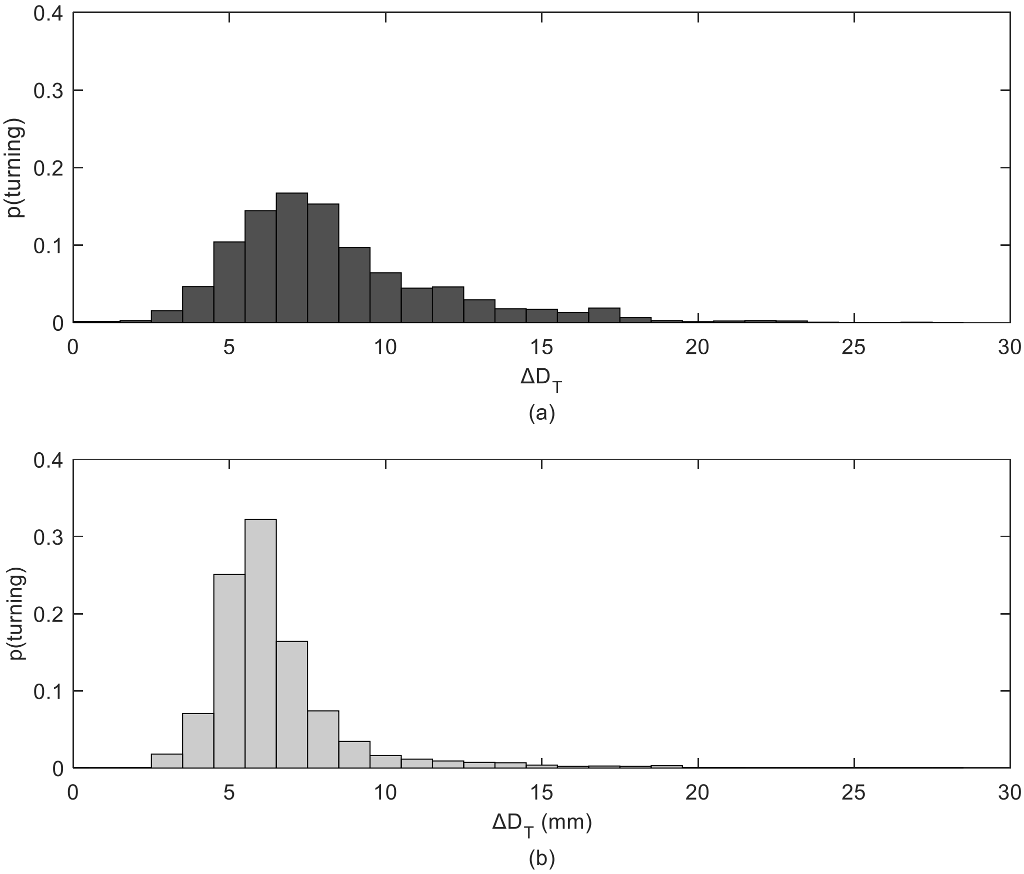


Figure 4.7: Histograms of the loss in diameter due to turning (ΔD_T) in a wheelset: (a) with damage and (b) without damage.

Proceeding in the same way as in Sharma (2016), Figure 4.7 was built using the relative frequency from past samples as an approximation of the transition probabilities, i.e.:

$$p(turning) = \frac{n_j}{N}, \tag{4.18}$$

in which n_j is the number of wheelsets that transit to a class j of diameter loss and N is the total number of wheelsets.

These transitions for the “turning” action in theory are schematically represented in Figure 4.8.

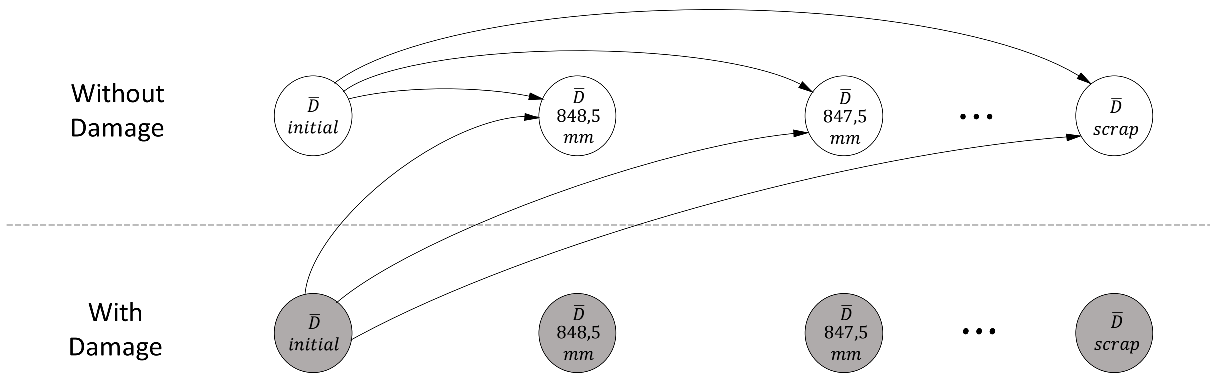


Figure 4.8: Transitions between states for the “turning” action if the wheel is in a state of $\bar{D}_{initial}$.

The construction of the MTM for the “turning action” is built up in diagonal expansions as the ones used in Equation (4.5), i.e. observing Figure 4.8, the transitions to next states are limited, which means that a transition from a state to another one with a great loss in the diameter does not happen at some point (according to Figure 4.7, it is defined 30 mm as the maximum loss in diameter possible). Therefore, regarding transitions from one state to the other ones, the probabilities are composed by zeros to states before the current one and zeros for states after the current one that the “turning” action “cannot reach”.

When a wheelset is turned, it goes back to a state where mileage since last turning (mst) is zero, and if it has damage it goes to a state without damage, since once the damage is detected it must be removed.

As it is not possible to turn a wheelset beyond the scrap diameter, when the wheelset is in a scrap diameter state, at some point of its mileage since last turning (mst), and the histograms of Figure 4.7 indicate diameter losses that go beyond the scrap diameter for that final state, what happens is that the probabilities of the remaining transitions are summed up becoming the probability value for the wheelset to stay at the final state, i.e. the scrap diameter, and similar to what is proposed in Equation (4.5).

Using the probability values withdrawn from Figure (4.7a), it is possible to compose the sub-transition matrix for the “turning” action from states without damage:

$$
P_{TND} = \left[p_{i,j}^{3}\right] = \begin{bmatrix}
p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & \cdots & p_{1,29}^{3} + p_{1,30}^{3} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & \cdots & p_{1,28}^{3} + p_{1,29}^{3} + p_{1,30}^{3} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \sum_{j=1}^{30} p_{1,j}^{3} = 1
\end{bmatrix}. \tag{4.19}
$$

In the same way, using now the probability values withdrawn from Figure (4.7b), it is possible to compose the sub-transition matrix for the “turning” action from states with damage:

$$
P_{TD} = \left[p_{i,j}^{3}\right] = \begin{bmatrix}
p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & p_{1,3}^{3} & \cdots & p_{1,30}^{3} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & p_{1,2}^{3} & \cdots & p_{1,29}^{3} + p_{1,30}^{3} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & p_{1,1}^{3} & \cdots & p_{1,28}^{3} + p_{1,29}^{3} + p_{1,30}^{3} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \sum_{j=1}^{30} p_{1,j}^{3} = 1
\end{bmatrix}. \tag{4.20}
$$

Finally, applying to the state space of Equation (4.3), the MTM when the “turning” action is chosen (P_3) is composed as follows:

$$
P_3 = \left\lfloor p_{i,j}^{3} \right\rfloor =
\begin{array}{c}
\begin{array}{ccccccc}
\leftarrow mst = 0\ miles & \leftarrow mst = 10\ miles & \leftarrow mst = 20\ miles & \cdots & \leftarrow mst = 240k\ miles & \leftarrow mst = 250k\ miles & \leftarrow States\ with\ damage
\end{array} \\
\begin{bmatrix}
P_{TND}^{(60\times60)} & 0 & 0 & 0 & 0 & 0 & 0 \\
P_{TND}^{(60\times60)} & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
P_{TND}^{(60\times60)} & 0 & 0 & 0 & 0 & 0 & 0 \\
P_{TD}^{(60\times60)} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^{(1620\times1620)}
\begin{array}{l}
\leftarrow mst = 0\ miles \\
\leftarrow mst = 10\ miles \\
\\
\vdots \\
\\
\leftarrow mst = 250k\ miles \\
\leftarrow States\ with\ damage
\end{array}
\end{array}
\qquad (4.21)
$$

The transitions used for the probability values in matrix P_3 can be seen in Appendix A3.

4.2 – Rewards/cost function

The MATLAB® Toolbox program chosen to solve this problem used a reward maximization function to derive the expected total discounted value rewards as in Equation (2.16). Therefore, the values used to represent the costs of the maintenance operations must be negative (Chadès et al. 2014).

To derive the rewards/costs function, a reward vector for each action chosen ($a = 1, 2, 3$) must be specified in a similar way of Equation (2.9).

The “do nothing” action ($a = 1$) does not hold any operational cost. However, it is important to guarantee, due to the state space constraints adopted, that when the wheelset reaches states of scrap diameter, mileage since last turning (mst) of 250 thousand miles or damaged states, other option different from “do nothing” is chosen. This is done by giving, to these critical states, cost values larger than the ones used in the remaining actions. For these states, it was assumed that values of -10 thousand monetary units should be assigned, as represented as follows:

$$q_i^1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ q_{60}^1(s_{60}) \\ 0 \\ \vdots \\ 0 \\ q_{120}^1(s_{120}) \\ 0 \\ \vdots \\ 0 \\ q_{120+60}^1(s_{120+60}) \\ 0 \\ \vdots \\ 0 \\ q_{1500}^1(s_{1500}) \\ q_{1501}^1(s_{1501}) \\ \vdots \\ q_{1560}^1(s_{1560}) \\ q_{1561}^1(s_{1561}) \\ \vdots \\ q_{1620}^1(s_{1620}) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -10000 \\ 0 \\ \vdots \\ 0 \\ -10000 \\ 0 \\ \vdots \\ 0 \\ -10000 \\ 0 \\ \vdots \\ 0 \\ -10000 \\ -10000 \\ \vdots \\ -10000 \\ -10000 \\ \vdots \\ -10000 \end{bmatrix} \begin{matrix} \\ \\ \\ \rightarrow \text{Scrap diameter} \\ \\ \\ \\ \rightarrow \text{Scrap diameter} \\ \\ \\ \\ \rightarrow \text{Scrap diameter} \\ \\ \\ \\ \rightarrow \text{Scrap diameter} \\ \left.\right\} mst = 250\ miles \\ \\ \left.\right\} \text{Damage} \end{matrix} \quad (4.22)$$

Based on the values presented in Andrade and Stow (2017b), it was considered for the "renewal" action ($a = 2$) an average value of -8000 monetary units, regardless of the state a wheelset is, and hence, the reward vector is as follows:

$$q_i^2 = \begin{bmatrix} q_1^2(s_1) \\ \vdots \\ q_{1620}^2(s_{1620}) \end{bmatrix} = \begin{bmatrix} -8000 \\ \vdots \\ -8000 \end{bmatrix}. \quad (4.23)$$

Moreover, and having in mind the values reported in Andrade and Stow (2017b), it was chosen a value of -400 monetary units to turn a wheel, independently of the wheelset state. However, similar to the case of the "do nothing" action, there are some critical states where a "renewal" action is needed, those are the cases when the scrap diameter is reached. And, therefore, for the "turning" action ($a = 3$) the reward vector is as follows:

$$q_i^3 = \begin{bmatrix} -400 \\ \vdots \\ -400 \\ q_{60}^3\,(s_{60}) \\ -400 \\ \vdots \\ -400 \\ q_{120}^3\,(s_{120}) \\ -400 \\ \vdots \\ -400 \\ q_{120+60}^3\,(s_{120+60}) \\ -400 \\ \vdots \\ -400 \\ q_{1500}^3\,(s_{1620}) \end{bmatrix} = \begin{bmatrix} -400 \\ \vdots \\ -400 \\ -10000 \\ -400 \\ \vdots \\ -400 \\ -10000 \\ -400 \\ \vdots \\ -400 \\ -10000 \\ -400 \\ \vdots \\ -400 \\ -10000 \end{bmatrix} . \begin{matrix} \\ \\ \\ \rightarrow \text{ Scrap diameter} \\ \\ \\ \\ \rightarrow \text{ Scrap diameter} \\ \\ \\ \\ \rightarrow \text{ Scrap diameter} \\ \\ \\ \\ \rightarrow \text{ Scrap diameter} \end{matrix} \qquad (4.24)$$

4.3 – Optimal policy

The MATLAB® Toolbox program derives the policy for this process in a decision vector as the one of Equation (2.14). Organizing the decision process in a graphic table for the damage and undamaged states with the evolution of the mileage since last turning (mst), it is possible to map a decision graphical table as represented in Figure 4.9.

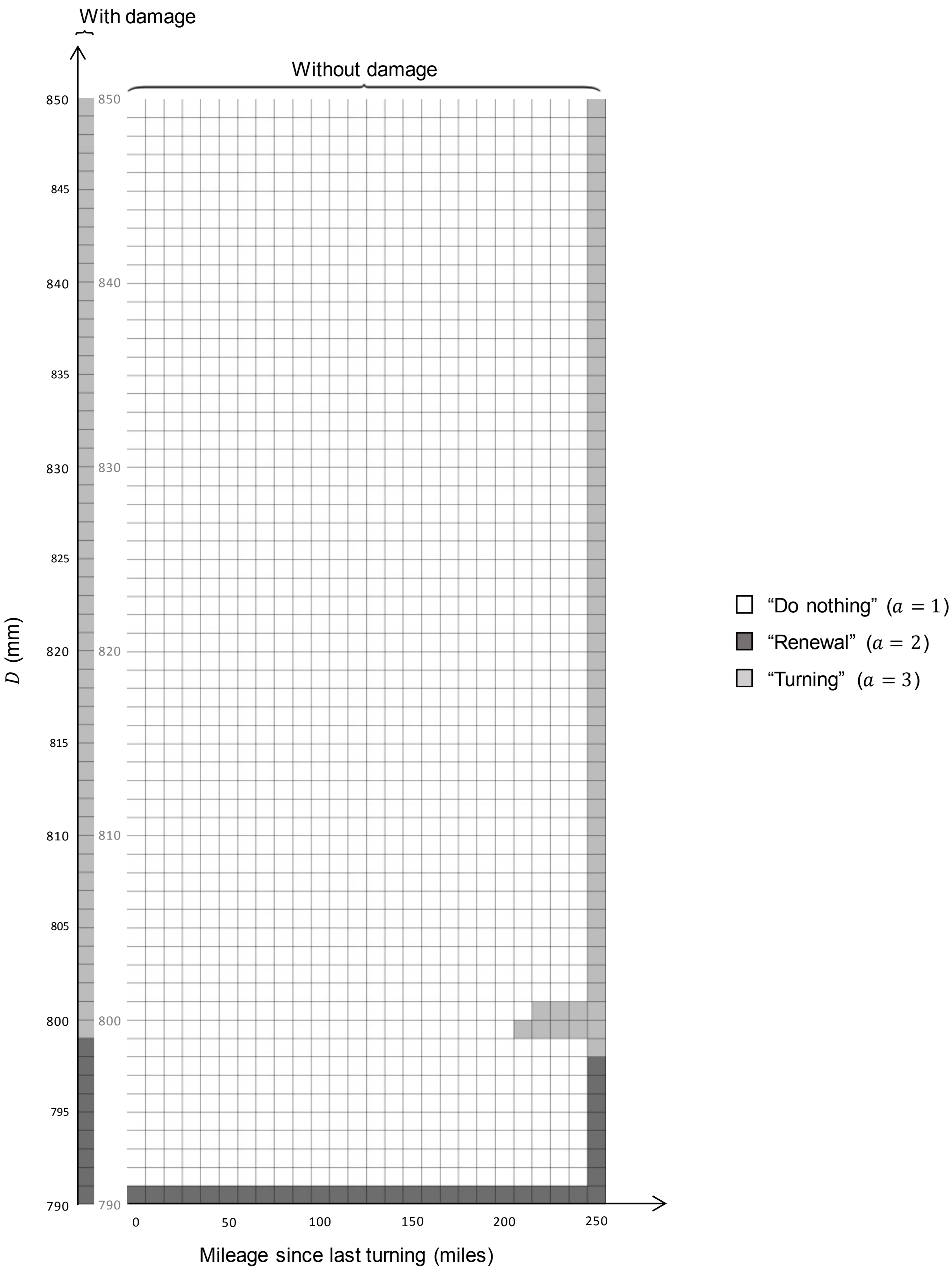


Figure 4.9: Map of decisions for wheelsets with and without damage with the evolution of the mileage since last turning (mst).

By analysing Figure 4.9, one can see that the transition probability methods adopted in section 4.1 and reward values chosen in section 4.2 resulted in actions that were intended: i) for the damaged wheelsets, only actions of "turning" or "renewal" are assigned, being the "renewal" actions left over for the last states where the “turning” action would go beyond the scrap diameter; ii) for the undamaged wheelsets, at states with a mileage of 250 thousand miles an action of “turning” or “renewal” is chosen, so the variable mst returns to zero, and “renewal” action is chosen for scrap diameter states and states where “turning” would go beyond it.

Figure 4.9 can then serve as a guideline for condition-based maintenance, i.e. depending on the diameter (D), mileage since last turning (mst) and whether or not damaged has occurred; it provides the optimal action that maximizes the total rewards (minimizes the total costs) for each defined wheelset state.

Chapter 5: Conclusions and Future Research

This final chapter provides the conclusions of the research, identifies some limitations and points out further steps of improvements and enhancements for the research here conducted.

5.1 – Conclusions

Wheelset maintenance is a procedure where wheelsets undergo a maintenance schedule to prevent wheelsets from failing when in service. Introducing condition-based planning and management using a MDP approach can be achieved, providing a more efficient method of managing the wheelsets assets than the traditional rules-based methods.

During this dissertation, the main problems were explained, improvements and procedures of the wheelsets inspection, degradation and maintenance. It was introduced the importance of the wheelset maintenance and inspection in the global context of the railway industry, presenting the main components of a wheelset, namely the ones that are more important for the inspection and consequently maintenance activities. The occurrence and evolution of the wheelsets degradation was explained and also a key factor for its comprehension was mentioned – the occurrence of damage.

Among all the possible approaches to model and solve this maintenance problem, it was chosen a data-driven model of decision and optimization based on the Markov Decision Problem (MDP). Its initial premises, basic elements, structural blocks and chain developments were presented.

This MDP model was then applied to a practical case of wheelset maintenance decision. It was defined the diameter change, the occurrence of damage and the mileage since last turning (or renewal) as the main indicators to control the process of maintenance and degradation of the wheelsets. The state space was divided into 1620 states according to the previous indicators and a set of three actions were defined: i) “do nothing”, ii) “renewal” and iii) “turning”. The decision process then was derived with the support of the MATLAB® MDP Toolbox (Chadès, 2014) and a useful map of decisions to support the decision-maker to take the best maintenance choice for each wheelset state was made.

Going into detail at the map of decisions of Figure (4.9) it is possible to conclude that an action of preventive maintenance (turning) would be advisable for railway wheelsets with a mileage since last turning between 210 and 240 thousand miles and a wheel diameter between 799 and 801 mm. In the remaining cases, the wheelset should run until the 250 thousand miles are completed between maintenance intervals or a damage has occurred.

5.2 – Limitations

Several limitations can be identified during the process that led to the present dissertation. First of all, the proposed modelling approach could not adopt the wheelset historical data from a Portuguese train operating company in useful time. This limitation was overcome by the use of past data and useful information from published papers (Andrade and Stow 2016, 2017a, 2017b).

Secondly, though the state space was sufficiently large to describe different diameters, mileage since turning and the occurrence of damage, it did not control for the evolution of the flange thickness and height as well as the angle inclination. Such limitations were not considered severe to the aim of the present thesis, but for further steps they would have to be included as the current standard limits these additional parameters. Nevertheless, from past experience, most of turning decisions are currently made based on mileage and/or the occurrence of damage and not due to these additional dimensions, and thus it is reasonable to argue that they might not affect to a great extent the optimal map that was made (Figure 4.9).

Moreover, an important limitation of the MDP approach is the estimation of the MTM and thus, the fact that such empirical approach is mainly data-driven, which represents an alternative way to vehicle dynamic simulations. Therefore, the conclusions drawn might be only applied to the data that was analysed and not immediately transferable to other case studies. In other words, the conclusions might be case specific and the MTM would have to be re-estimated (using Survival statistical models, for example) to feed this new case study applied to the Portuguese train operating company.

Finally, the proposed analysis did not take into account the uncertainty associated with inspection activities themselves, in particular using laser or human-based inspection procedures. The analysis of historical data would have to take this into account, i.e. condition data was collected with different precisions over time.

5.3 – Future research

Future research would have to overcome the points identified above as limitations of the present work. As MDPs provide a powerful framework for the solution of problems of maintenance and inspection, and the optimization of these processes has received considerable attention along the years, there are some extensions of the MDPs, such as Latent or Hidden Markov Decision Processes (Madanat 1993 and Madanat and Ben-Akiva 1994) or Partially Observable Markov Decision Processes (POMDP) (Papakonstantinou and Shinozuka 2014b and Young 2013) that might provide useful steps for further research. These approaches consider hidden observation variables or belief states, since they assume that it is not possible to know exactly the real state/condition of a wheelset because of the uncertainty of the equipment used in the inspections. Therefore, the value functions of these optimization processes deal with the uncertainty and these are more flexible processes. To accomplish that, observability matrices that contaminate with noise the current MTMs would have to be estimated, using the different precisions of the inspection equipment. Ideally, one would assess the optimal value of the MDP with human-based inspection and the optimal value with laser inspection and compare them to estimate the economic benefits of introducing laser inspection.

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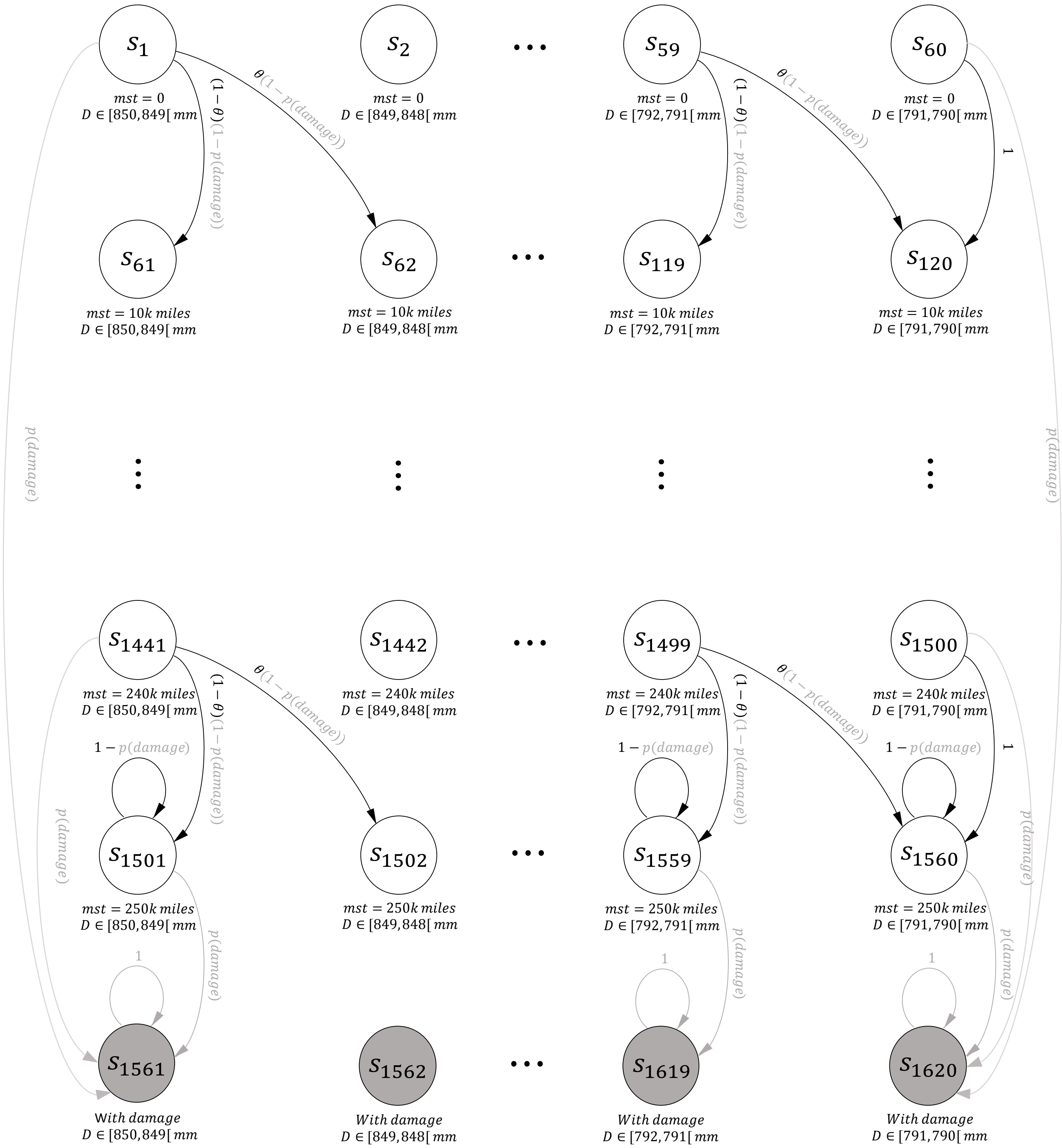
Appendix

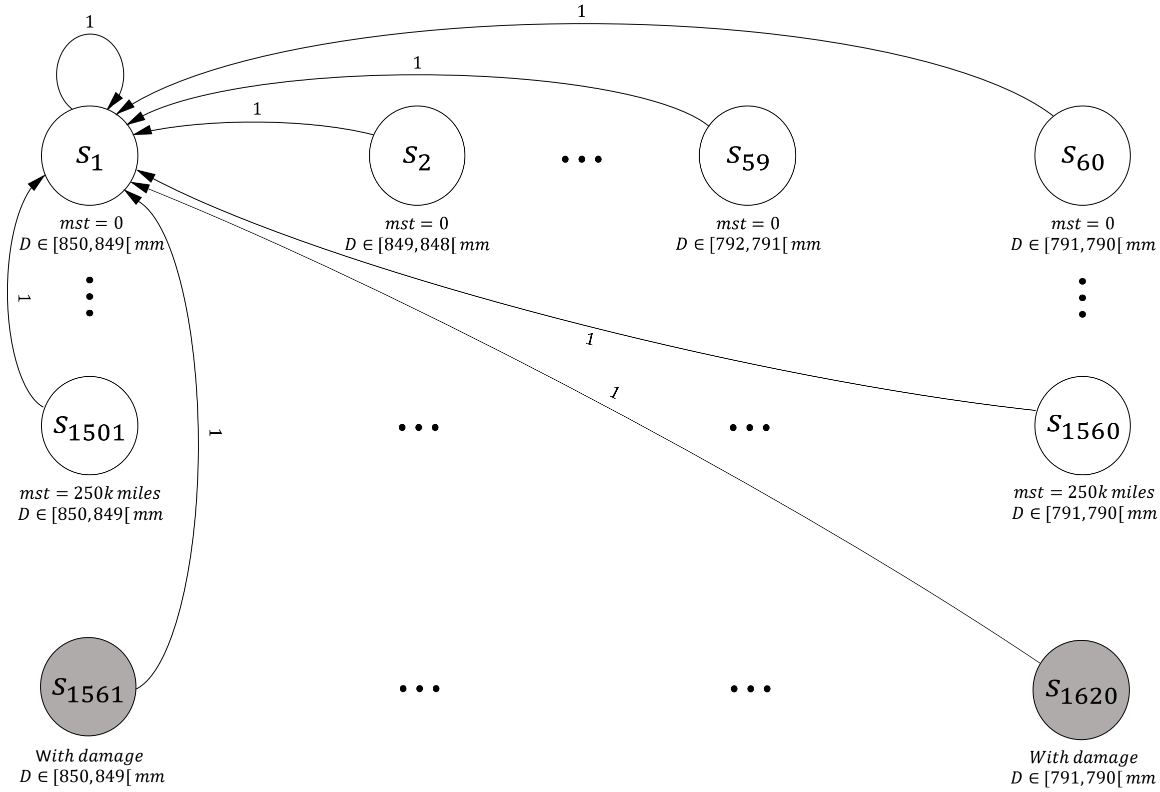
A1 State Space division and transition probabilities for the “do nothing” action ($a = 1$)

Note: grey colour represents the effect of damage.



A2 State Space division and transition probabilities for the “renewal” action ($a = 2$)

Note: grey colour represents the effect of damage.



A3 State Space division and transition probabilities for the “turning” action ($a = 3$)

Note: grey colour represents the effect of damage.